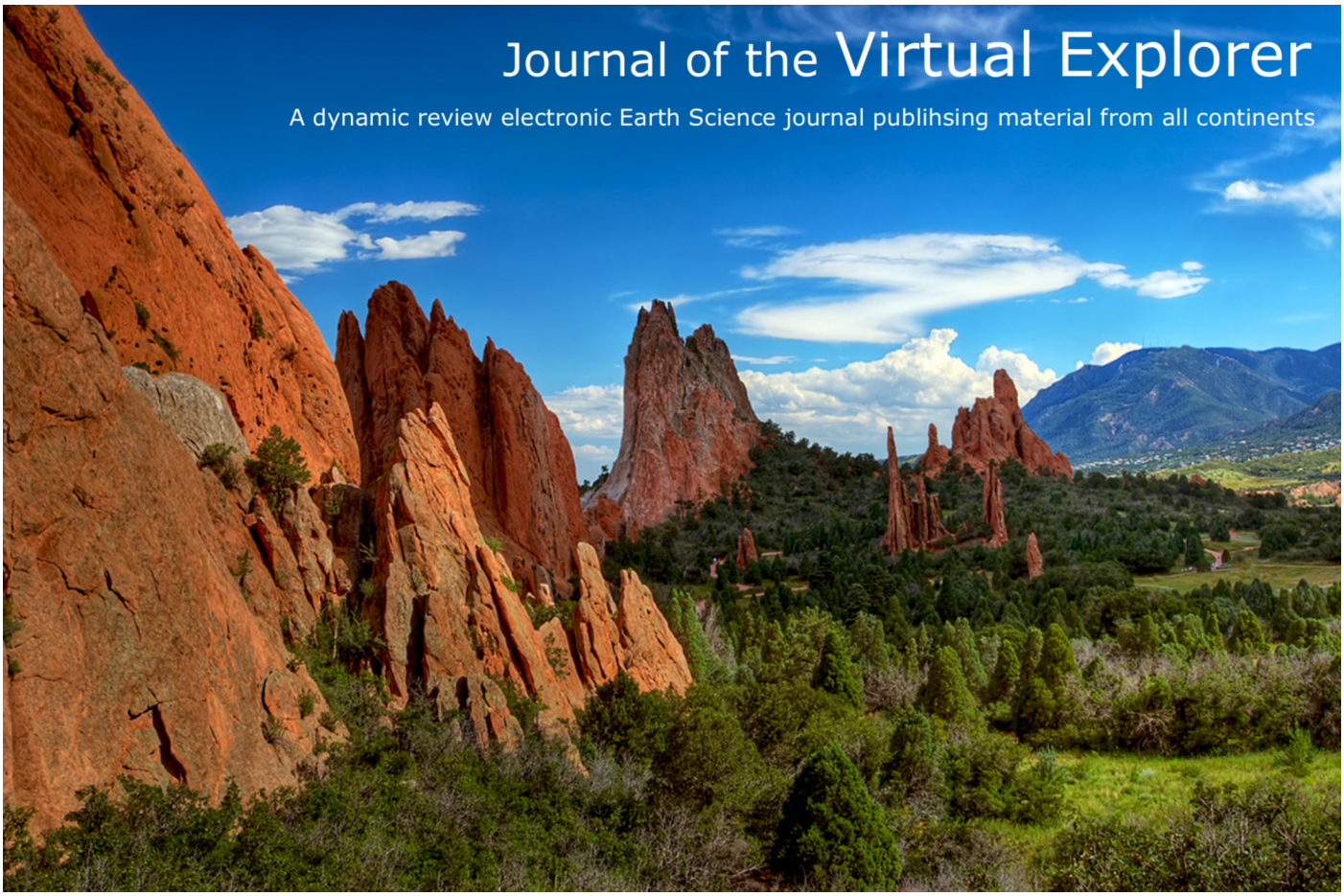

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Journal of the Virtual Explorer, Electronic Edition, ISSN 1441-8142, volume **21**, paper 1
In: (Eds.) Daniel Köhn and Declan De Paor, General Contributions, 2006.

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Abstract: Most natural folds are complex fractal folds. The fractal dimension of a fold can indicate its complexity and its ability (or degree) to fill a space. The folding and its associated fractal dimensions are affected by many factors; e.g. thickness, viscosity, multiple layers, layer parallel shortening, strain softening, initial perturbation, other instabilities, preferred wavelength, non-linear material and layer anisotropy, even the bond between layer and matrix, and many other unknown factors in which the thickness and viscosity of the layer are important. Assuming only two factors (viscosity and thickness) control the complexity of folds in the buckling of a single layer, a formula is derived in this paper to represent the relation between fractal dimension (D) and rheology properties including the layer thickness (h) and viscosity (μ). Information about rheology can potentially be gained from analyzing the fractal geometry of folds. The rheological formula of fractal folds shows that the fractal dimension of folds is affected by the coupling of thickness (h) and viscosity (μ) of a single layer. In conclusion, a thicker layer with higher viscosity may more easily develop more complex folds with higher fractal dimensions, filling more space than a thin layer with lower viscosity. A higher contrast of viscosities or thickness between two layers can yield a larger difference (D_1-D_2) between two fractal folds. This paper gives a theorical explanation for Ramberg's experiment.

http://virtualexplorer.com.au/article/2006/146/rheological-factors-and-fractal-folds

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Introduction

Though various methods of describing or characterizing the geometrical characteristics of folds have been proposed (Fleuty, 1964; Ramsay, 1967; Wilson, 1967; Elliot, 1968; Stabler, 1968; Hudleston, 1973a; Fletcher, 1977, 1979; Ramsay & Huber, 1987; Twiss, 1988; Johnson & Pfaff, 1989; Bastida, 1993; Grujic et al., 2002; Srivastava and Lisle, 2004), the mechanisms of folding continue to be difficult to study in the field of structural geology, as the folding deformation is rheologically dependant. This is far more complex than simple elastic deformation, highly variable fold geometries are generated depending on the rheological properties of what are usually non-linear materials.

Single-layer buckling has attracted the attention of theoreticians and experimentalists for nearly half a century. It's understanding is considered to be a key step to interpreting the complexity of natural folding and deformation. Ramberg (1960, 1964) and Biot (1961, 1964) proposed the theoretical and physical models to explain the rheological mechanism of single-layer buckle folding. The initial perturbation affects the buckling of a single-layer (Mancktelow and Abbassi, 1992; Zhang et al., 1996, 2000). The stress and strain are considered to be important factors affecting the buckling (Treagus and Sokoutis, 1992; Jeng et al., 2002). Lan and Huddleston (1996) employed finite elements to study the sharpness and development of buckle folds by curvature and finite strain, and suggested that two basic material properties affecting fold shape include non-linearity and anisotropy. They also discussed the rheological properties in numerical models (Huddleston & Lan, 1993, 1994; Lan & Huddleston, 1991, 1995, 1996). These studies are only concerned with simple buckling and agree that viscosity and thickness are the two most important factors affecting the buckling of a single-layer, but the coupling of two factors in complex fractal folds has not yet been studied.

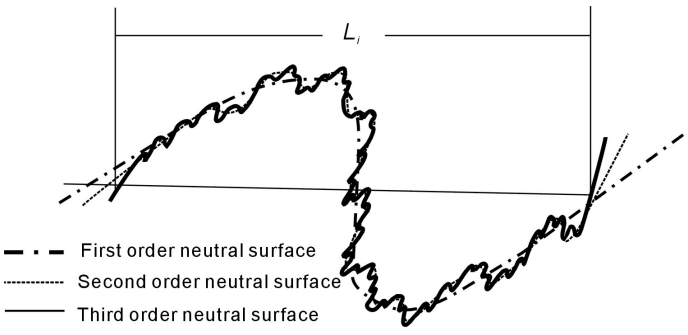
Many factors affect folding and the fold complexity; e.g. the thickness, viscosity, multiple layers, layer parallel shortening, strain softening, initial perturbation, other instabilities, preferred wavelength, non-linear material and layer anisotropy, even the bond between layer and matrix, and many other unknown factors (Chapple, 1969; Fletcher, 1974,1977; Huddleston, 1973b; Abbassi and Mancktelow, 1990; Huddleston and Lan, 1994a, 1994b; Ramberg, 1964; Ramsay and Huber, 1987; Treagus, 1973, 1981, 1992; Zhang et al., 1996; Mancktelow, 1999), in which the thickness and viscosity of any layer is important. The question to be asked is: How does the coupling of thickness and viscosity affect the complexity of folds?

In this paper, we combine fractal theory with rheology to investigate how the fractal dimension (D) of complex folds is affected by the coupling of viscosity and thickness in single-layer buckling.

Natural complex fractal folds

Most natural folds are not as simple as those generated by physical and numerical models, producing folds that are smooth and linear. Natural folds are usually complex, nonlinear and composed of many orders of folding of different magnitudes. The anticlinorium and synclinorium commonly developed in any orogenic belt are self-similar, called a fractal pattern. In the field, the so-called 'S','Z','M' and 'W' parasitic folds are observable from different parts of some outcrop-scale folds. This scaling property of the folds ranges from tens of kilometers of compound folds in orogenic belts, to folds in outcrop and sample scale, and even to micron scale, indicating that the natural folds are of the fractal geometry (Wu, 1993). The anticlinorium and synclinorium are formed by superimposing smaller scale folds. The order of a complex folds is distinguished from the neutral surfaces or the enveloping surfaces of the fold (Turner & Weiss, 1963, Ramsay, 1967, Ramsay & Huber, 1987). Natural folds commonly have three orders (Figure 1).

Figure 1. Fold with three orders identified



The model showing a fold with three orders identified by the neutral surfaces of the fold

Self-similarity of structure is an important characteristic of fractal geometries, in which any portion of the system is a scaled-down version of the whole (Mandelbrot, 1983). A feature of a fractal geometry is that the relative

numbers of large and small elements remain the same at all scales between the upper and lower fractal dimension, which is simply derived from the power-law exponent on a plot of log ruler (l) vs. log cumulative number(N) (Formula 1) (Mandelbrot, 1983).

Most natural folds are complex, variable, and develop many fold orders with self-similarity in fractal patterns. They can therefore be described quantitively by fractal geometry (Wu, 1993, Hou, 1998). Complex folds can be simulated by fractal interpolation and classified into different populations by their complexity relative to the fractal dimension D (Hou, 1998).

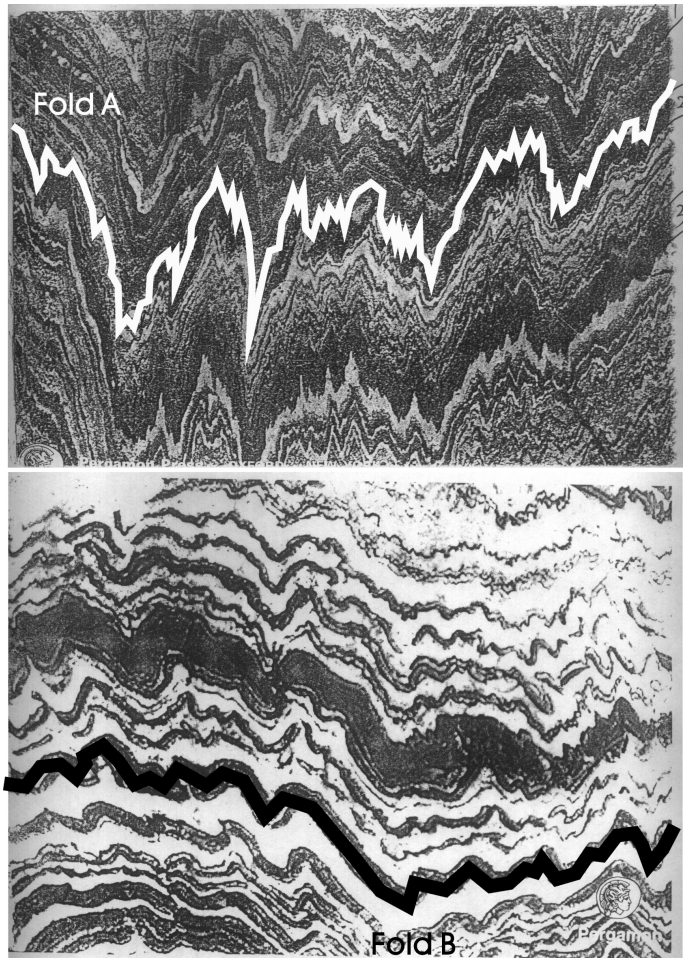
$$D = -\frac{lgN(l)}{lgl}$$

Where D is the fractal dimension of the fold, with a value between 1.0 and 2.0 in two dimensional space.

A fractal fold profile is, defined by Mandelbrot (1983), as a curve with a fractal dimension (D) greater than its topological dimension. A topological dimension is an integer such that a discrete point has a topological dimension of zero, a curve of 1, and so on. The fractal dimension of a fold profile describes how much of the two-dimensional space it fills. The fold with bigger amplitude and more orders of folding means it fill more of the space, and so its fractal dimension is larger. If a fold profile consists of a simple line without any curve, its fractal dimension D = 1. If a fold profile is so complicated that it almost fills the entire space, its fractal dimension D will be close to 2. Thus, most fractal fold profiles have a fractal dimension 1 < D < 2 (Wu, 1993; Hou, 1998). Thus, the complexity of a fold can be described by the fractal dimension (D), with more complex folds usually having a larger value of D (e.g. Figure 2). Obviously, the fold with D of 1.48 (correlation coefficient=0.996) is more complicated than the fold with D of 1.38 (correlation coefficient=0.996).

There are two basic types of fractals: one is self-similar, the other is self-affine (Mandelbrot, 1985), and both are statistically defined (Power & Tullis, 1991). The self-similar fractal fold has a constant amplitude-to-wavelength ratio, while an important aspect of a self-affine fractal is that the self similarity is not universal and changes with scale (Wu, 1993, 1995). Most natural fractal folds are self-affine fractals.

Figure 2. Two fractal folds with different complexities



Two fractal folds with different complexities (different fractal dimensions)

A: Folds in banded hornblende gneiss in the core of the Maggjia Nappe from Fusio, Ticino, Switzerland;

B: Folds of calc-silicate layers in marble,Neoproterozoic deposits in the Khan River Gorge, Central Namibia

Rheological factors of complex fractal folds

Complex buckle folds with multiple orders can develop in isolated stiff layers in a less stiff matrix, a multilayered package where individual layers vary in composition and thickness, and also in homogeneous but anisotropic materials. Single-layer folds are much less variable than multilayered folds, so in this study, we restrict our investigation to complex folds developed by the buckling of a single isolated viscous layer in a less viscous matrix.

A very simple geometry is considered in this model, namely a single isolated layer within a homogeneous matrix deformed under conditions of pure shear, with the shortening direction parallel to the layer. This geometry

is similar to that employed in many previous experiments using both elastic and viscous materials (e.g. Biot et al., 1964, Ramberg, 1964; Huddleston, 1973b), also, approximately symmetric single-layer folds are fairly common in nature (Sherwin & Chapple, 1968, Cobbold et al.1971). Most natural folds are highly variable and complicated due to the fact that they usually result from the rheological deformation on a geological time scale. For complex folds, the rheological deformation forms the major component while elastic deformation is negligible (Biot et al., 1964, Ramsay, 1967).

Many factors affect fold shape and development. There are too many unknowns to deduce rheological properties unambiguously from populations of natural folds. If a reasonable assumption about the form of the initial wavelength spectra is made, if strain in the competent layers can be measured independently, and if anisotropy is unimportant, estimates of rheological parameters can be made (Fletcher, 1974). Based on the experimental and theoretical rheology models, Biot (1964) and Ramberg (1964) proposed the theory of buckle folding, which predicts that, for a system consisting of an isolated stiff viscous layer in a less viscous matrix subject to layer-parallel pure shear, all harmonic components that are present in the initial irregularities of the layer interfaces will become amplified, with a maximum rate of growth occurring for the 'dominant wavelength' (Biot, 1964, Ramberg, 1964, Fletcher, 1974). For Newtonian materials, the dominant wavelength is a function of the thickness and the viscosity ratio, of which the expression can be given as:

$$L_i = 2\pi h_i \sqrt[3]{\frac{\mu_i}{6\mu_0}}$$

Where L_i is the dominant wavelength, μ_i is the viscosity of the layer i, μ_0 is the viscosity of matrix, and hi is the thickness of layer i. The formula (2) also represents that, for Newtonian materials, the ratio of dominant wavelength to thickness is a function only of the ratio of viscosity of the stiff layer to the viscosity of the matrix.

The bigger the viscosity contrast between a stiff layer and the matrix is, the faster initial folds will grow. Two separated layers develop different folds during the layer parallel shortening in the same matrix, and seem to undergo different shortening. In fact, the layers and matrix undergo the same quantitive layer parallel shortening, but the contributions of layer parallel shortening and buckle folding are varied in different layers with other viscosities and thicknesses (Ramberg, 1964, Ramsay & Huber, 1987).

Here, we assume a model where the complexity of fractal folds is only affected by the viscosity and thickness in a single-layer buckling scenario. In comparison, two layers with different viscosities and thicknesses are each in a matrix with the same viscosity, given in the models (Figure 3).

In a matrix with viscosity of μ_0, layer 1 with viscosity ofμ1 and thickness of h_1, and layer 2 with viscosity of μ_2 and thickness of h_2, both develop fractal buckle folds 1 and 2 at the same strain, their dominant wavelength are L_1 and L_2, which are given by:

$$L_1 = 2\pi h_1 \sqrt[3]{\frac{\mu_1}{6\mu_0}}$$

$$L_2 = 2\pi h_2 \sqrt[3]{\frac{\mu_2}{6\mu_0}}$$

Based on the fractal theory (Mandelbrot, 1983, 1985, Falconer, 1990), the relationship between the fold arc Li(l) measured by the scale factor l and the dominant wavelength Li is given by:

$$\frac{L_i(l)}{L_i} = C_i l^{(l-D_i)}$$

Where L i, the dominant wavelength of fold i, is a reference length for normalization (Figure 1), Di is the fractal dimension of fold i, Ci is constant in equation (5).

For fold 1, the equation (6) is given by

$$\frac{L_1(l)}{L_1} = C_1 l^{(l-D_1)}$$

For fold 2, the equation (7) is given by

$$\frac{L_2(l)}{L_2} = C_2 l^{(l-D_2)}$$

The equation (8) is given from equation (6) and (7)

$$\frac{L_2(l)}{L_1(l)} . \frac{L_1}{L_2} = C(l) \frac{L_1}{L_2} = C l^{(D_1 - D_2)}$$

Where l is the scale factor, $C = C_2 / C_1$, $C(l) = L_2(l) / C_1(l)$, L1 is the dominant wavelength of

fold 1, L2 is the dominant wavelength of fold 2, L1 (l) is the arc length of fold 1 measured with respect to the scale factor l, L2 (l) is the arc length of fold 2 measured with respect to the scale factor l. D1 and D2 are the dimensions of fold 1 and fold 2 respectively.

Equation (9) is obtained from the equation (3) and (4):

$$\frac{L_1}{L_2} = \frac{2\pi h_1 \sqrt[3]{\frac{\mu_1}{6\mu_0}}}{2\pi h_2 \sqrt[3]{\frac{\mu_2}{6\mu_0}}} = \frac{h_1}{h_2}\sqrt[3]{\frac{\mu_1}{\mu_2}}$$

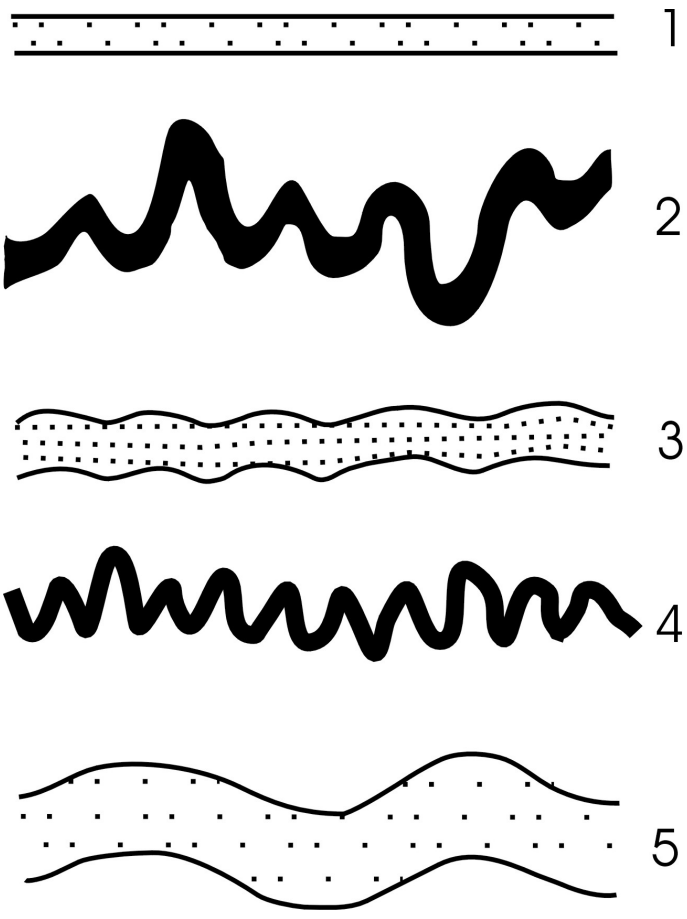
thus, equation (10) is obtained from (8) and (9):

$$C(l)\left(\frac{h_1}{h_2}\sqrt[3]{\frac{\mu_1}{\mu_2}}\right) = Cl^{(D_1-D_2)}$$

or

$$Cl^{(D_1-D_2)} = C(l)\frac{h_1}{h_2}\sqrt[3]{\frac{\mu_1}{\mu_2}}$$

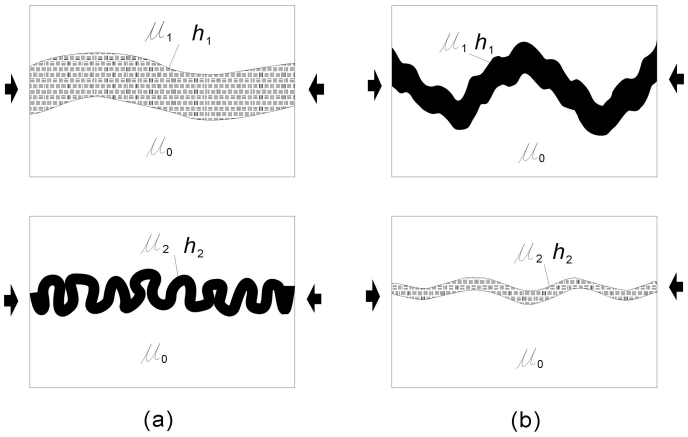
Figure 3. Four possible models of buckling in an incompetent matrix.



(a) Layer 1: long wave folding with more contribution from shortening. layer 2: easily develops snake folds. Layer 2 is more complex than layer 1 in where h1 > h2, μ1 >μ0 and μ2 >>μ0.

(b) Layer 1: easily develops complex folding with different orders, layer 2: small wave folding with more contribution from shortening. Layer 1 is more complex than layer where h1 > h2, μ1 >> μ0 and μ2 > μ0.

Figure 4. The results of buckling experiment



The results of buckling experiment in different rheological conditions where strain is constant (after Ramberg, 1964).

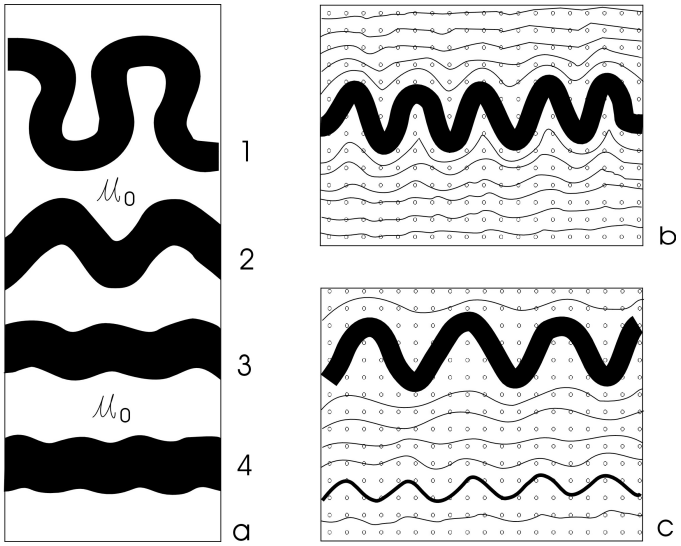
μ4 >μ2 >μ3 >μ5 >μ1 >μ0, h5>h3>h2>h1>h4

(μ1, μ2, μ3, μ4, μ5 and μ0 are the viscosities of layer 1, 2, 3, 4, 5 and matrix respectively, h1, h2, h3, h4 and h5 are thicknesses of layers 1, 2, 3, 4, and 5 respectively)

Based on the characteristics of fractal geometries, the dimension D is not related to the scale factor l, so C(l) is a variable constant related to l, but doesn't change the fractal dimension D. The difference (D1-D2) representing the complexity contrast between two fractal folds is controlled by the ratio of viscosities (μ1/μ2) and ratio of thicknesses (h1/h2) between two folds. The larger contrast of viscosities or thickness between two layers can yield greater difference (D1-D2) between two fractal folds. In nature, the viscosity contrast (μ1/μ2) is commonly much bigger than the ratio of two thicknesses (h1/h2). The viscosity contrast, therefore, plays a more important role in affecting the complexity of buckling folds of a single-layer than the thickness if only two factors (viscosity and thickness) are considered in the buckling. If the viscosity contrast is large, the fold will grow in amplitude at a rapid rate. The layers 2 and 4 are competent layers, developing more complex fractal folds than other incompetent layers (Figure 4). At a small or moderate viscosity contrast, there is a rapid increase in layer thickness by layer-parallel shortening and a slow growth of

fold amplitude (Figure 5a). In the same strain condition, the competent layer experiences more buckle folding than layer parallel shortening, while the incompetent layer experiences more layer parallel shortening than buckle folding (Figure 5 a, b and c).

Figure 5. Viscosity and thickness constraints



Viscosity and thickness constraints on the shape of fractal folds (after Ramsay & Huber, 1987).

(a) Folds with various viscosities ($\mu1>\mu2>\mu3>\mu4>\mu0$, h1=h2=h3=h4), a higher viscosity contrast in a layer allows more buckle folding, $\mu1/\mu0 > 50$, $50 > \mu2 / \mu0 > \mu3 / \mu0 > 10$, $\mu4 / \mu0 < 10$.

(b) Thicker competent layer (black) with greater amplitude and wavelength.

(c) The thicker of the two competent layers (black) has greater amplitude and wavelength.

The thickness of a layer also affects the complexity and harmony of folds. The thickness of a single-layer can control the shape of folds. Two single-layers with a higher thickness ratio are more disharmonic (Castro & Cashman, 1999). Assuming the same strain condition is applied, the thicker layer can develop a larger dominant wavelength and have more chance of developing complex folds with higher fractal dimensions than a thin layer, while the thin layer only develops small wavelengths (Figure 5 b and c). Layer 3 and 5 develop more complex folds than layer 1 (Figure 4). In general, a thicker layer with higher viscosity more easily develops more complex folds with larger fractal dimensions, filling more space than a thin layer with lower viscosity (Figure 3, comparing layer 1 with layer 2).

The equation (10) suggests that the coupling of at least two factors (viscosity and thickness) can produce different fold shapes and make complex folds. Otherwise, the fractal dimension difference between two folds can indicate the harmonic degree. Two folds are harmonic if the difference of two fractal dimensions (D1-D2) is zero. The larger the difference between the fractal dimensions of two folds is, the larger the disharmonic degree. This is supported by folds observed in the field (Castro & Cashman, 1999).

The formula (10) gives a theorical explanation to the experiment shown in Figure 4 and the formula is expanded to explain the complexity of the folds. In fact, the complexity of a fractal fold is affected by both the viscosity and thickness of a layer. This is a coupling effect, for example: a thick soft layer cannot form a complex fold and is mostly undergoes layer-parallel shortened. The complex folds with large fractal dimensions result from the coupling of greater thickness and higher viscosity of a single-layer. In the buckle folding experiment, a thin competent layer can easily develop folds with a small wavelength, and a thick layer can develop folding with a big dominant wavelength. Therefore the thick competent layer can develop complex folds with varying characteristic wavelengths in different orders of folding. The viscosity and the thickness contrast between layer and matrix are very important in affecting the complexity of fractal folds. In nature, a thick layer with a low viscosity contrast usually develops layer-parallel shortening, the thin layer with high viscosity usually develops ptygmatic folds, and commonly the thick, competent layer develops complex folds including many orders. In conclusion, the coupling of two rock properties (viscosity and thickness) can produce complex folds and affect the fractal dimension of complex folds.

Conclusion

Natural folds are too complex to know which rheological model best suits them due to the uncertainty about the mechanical flow laws appropriate to complex geological conditions. Fractal geometry is an effective theory to describe the complexity of folds. This paper combines fractals and rheology to study the rheological mechanism of buckle folding and derives the formula for this relationship, also suggesting that the viscosity and thickness are very important factors affecting the complexity of folds. The coupling of two factors is only studied in

qualitative analysis, and is not yet confirmed by the limitation of two factors in quantitative analysis. A quantitative analysis should be investigated to understand how these two factors work in affecting the complexity of fractal folds in future. Otherwise, the complexity of folding is not controlled by a unique rheological factor, but is affected by the integration of the various rheological factors. The paper simply tries to give a theoretical explanation for Ramberg's experiment.

In fact, many rheological factors affect the complexity of folds, for example, layer parallel shortening, strain softening, initial perturbation, other instabilities, preferred wavelength, non-linear material, layer anisotropy, even the bond between layer and matrix, and many other unknown factors. In the models suggested in this paper, only viscosity and thickness are assumed to affect the fractal dimensions of single-layer buckling. The fractal dimension of single-layer buckling is influenced by the coupling of viscosity and thickness in the models. In conclusion, a thicker layer with higher viscosity more easily develops more complex folds with greater fractal dimensions, filling more space than thin layer with lower viscosity. Further experiments need be done in various conditions to investigate the rheological factors which affect the complexity of folds. This opens the potential for future work in the study of complex fold rheology.

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References

Abbassi, M. R. & Mancktelow, N. S., 1990. The effect of initial perturbation shape and symmetry on fold development. Journal of Structural Geology 12, 273-282.

Bastida, F., 1993. A new method for the geometrical classification of large data sets of folds. Journal of Structural Geology 14, 85-104.

Biot, M.A., 1961. Theory of folding of stratified viscoelastic media and its implication in tectonics and orogenesis. Geological Society of America Bulletin 72, 1595-1620.

Biot, M.A., 1964. Theory of viscous buckling of multilayered fluids undergoing finite strain, Physics Fluids 7,855-859

Castro, J. & Cashman, K. V., 1999. Constraints on rheology of obsidian lavas based on mesoscopic folds. Journal of Structural Geology 21, 807-819.

Chapple, W. M., 1968. A mathematical theory of finite-amplitude rock-folding. Bull. Geol. Soc. Am. 79, 47-68.

Chapple, W. M., 1969. Folding shape and rheology: the folding of an isolated viscous-plastic layer. Tectonophysics 7, 97-116.

Cobbold, P. R., Cosgrove, J. W. Summers, J. M., 1971. Development of internal structures. Tectonophysics 12, 23-53.

Elliot, D., 1968. Interpretation of fold geometry from Lineation isogonic maps, J.Geol.76.171-190

Falconer, K., 1990. Fractal Geometry: Mathematical Foundations and Applications. Wiley, Chichester.

Fleuty, M.J., 1964.The description of folds, Proc.Geol.Assoc.London 75,461-492

Fletcher, R. C., 1974. Wavelength selection in the folding of a single layer with power-law rheology. Am. J. Sci. 274, 1029-1043.

Fletcher, R. C., 1977. Folding of a single viscous layer: exact infinitesimal-amplitude solution. Tectonophysics 39, 593-606.

Fletcher, R. C., 1979. The shape of single-layer folds at small but finite amplitude. Tectonophysics 60, 77-87.

Grujic, D., Walter, T. R., Gartner, H., 2002. Shape and structure of (analogue models of) refolded layers. Journals of Structural Geology 24, 1313-1326.

Hou, Guiting, 1998. Fractal simulation and classification of folds. Acta Geologica Sinica .Vol.72, 2,217-223

Huddleston, P. J. & Lan, L., 1993. Information of shapes. Journal of Structural Geology 15,253-264.

Huddleston, P. J. & Lan, L., 1994a. Rheological controls on the shapes of single-layer folds. Journal of Structural Geology 16, 1007-1021.

Huddleston, P. J. & Lan, L., 1994b. The effect of anisotropy on buckle fold development. Geol. Soc. Of Am. Abstracts with Programs 26, A315

Huddleston, P. J., 1973a. .Fold morphology and some geometrical implications of theories of fold development. Tectonophysics 16, 1-46.

Huddleston, P. J., 1973b. An analysis of “single layer” folds developed experimentally in viscous media.Tectonophysics 16,189-214

Jeng, F. F., Lin, M. L., Lai, Y. C. et al., 2002. Influence of strain rate on buckle folding of an elasto-viscous single layer. Journal of Structural Geology 24, 501-516.

Johnson, A. M. & Pfaff, V. J., 1989. Parallel, similar and constrained folds. Eng. Geol. 27, 115-180.

Lan, L. & Huddleston, P. J., 1991. Finite element models of buckle folds in non-lieaner materials.Tectonophysics, 199,1-12

Lan, L. & Huddleston, P. J., 1995. A method of estimating the stress exponent in the flow law in rocks using fold shape. Pure Appl. Geophys. 145,620-633

Lan, L. & Huddleston, P. J., 1996. Rock rheology and sharpness of folds in single layers. Journal of Structural Geology 18,925-931

Mancktelow, N.S. & Abbassi, M. R., 1992. Single layer buckle folding in non-linear materials-II. Comparison between theory and experiment. J. Stru. Geol. 14, 105-120.

Mancktelow, N. S., 1999. Finite-element modeling of single-layer folding in elasto-viscous materials: the effect of initial perturbation geometry. Journal of Structural Geology 21, 161-177.

Mandelbrot, B. B., 1983.The fractal Geometry of Nature. Freeman, New York

Mandelbrot, B. B., 1985.Self-affine fractals and fractal dimension, Physica Scripta 32,257-260

Power, W. L. & Tullis, T.E., 1991. Euclidean and fractal models for the description of rock surface roughness. J.Geophy. Res.96, 415-424

Ramberg, H., 1960. Relationship between length of arc and thickness of ptygmatically folded veins. American Journal of Science 258, 36-46.

Ramberg, H., 1964. Selective buckling of composite layers with contrasted rheological properties. Tectonophysics 1, 307-341.

Ramsay, J. G., 1967. Folding and Fracturing of Rocks. McGraw Hill, New York.

Ramsay, J. G. and Huber, M. I., 1987. The techniques of modern structural geology.Vol.2: Folds and Fractures. Academic Press, London.

Sherwin, J. & Chapple, W. M., 1968. Wavelengths of single layer folds: a comparison between theory and observation. Am. J. Sci. 266, 167-179.

Srivastava, D. C., Lisle, R. J., 2004. Rapid analysis of fold shape using Bézier curves. Journal of Structural Geology 26, 1553-1559.

Stabler, C. L.1968. Simplified Fourier analysis of fold shapes.Tectonophysics 6,343-350

Treagus, S. H., 1973. Buckling instability of a viscous single-layer system, oblique to the principal compression. Tectonophysics, 19, 271-289.

Treagus, S. H., 1981. A theory of stress and strain variations in viscous layers, and its geological implications. Tectonophysics 72, 75-103.

Treagus, S. H. & Sokoutis, D., 1992. Laboratory modeling of strain variation across rheological boundaries. Journal of Structural Geology 14, 405-424.

Turner, F.J. and Weiss, L.E.1963.Structural analysis of metamorphic tectonics, McGraw-Hill, New York and London, 545

Wilson, G., 1967. The geometry of cylindrical and conical folds. Proc. Geol. Ass. 78, 179-200.

Wu, S.1993.Fractal analysis of fracture patterns using the standard box-counting technique: valid and invalid methodologies. Journal of Structural Geology, 15, 1497-1507

Wu, S.1995.Fractal strain distribution and its implications for cross-section balancing: Reply. Journal of Structural Geology 15,1509-1512

Zhang, Y., Hobbs, B. E., Ord, A., Muhlhaus, H. B., 1996. Computer simulation of single-layer buckling. Journal of Structural Geology 18, 643-655.

Zhang, Y., Mancktelow, N. S., Hobbs, B. E., Ord, A., Muhlhaus, H. B. 2000. Numerical modeling of single-layer folding: clarification of an issue regarding the possible effect of computer codes and the influence of initial irregularities. Journal of Structural Geology 22, 1511-1522.