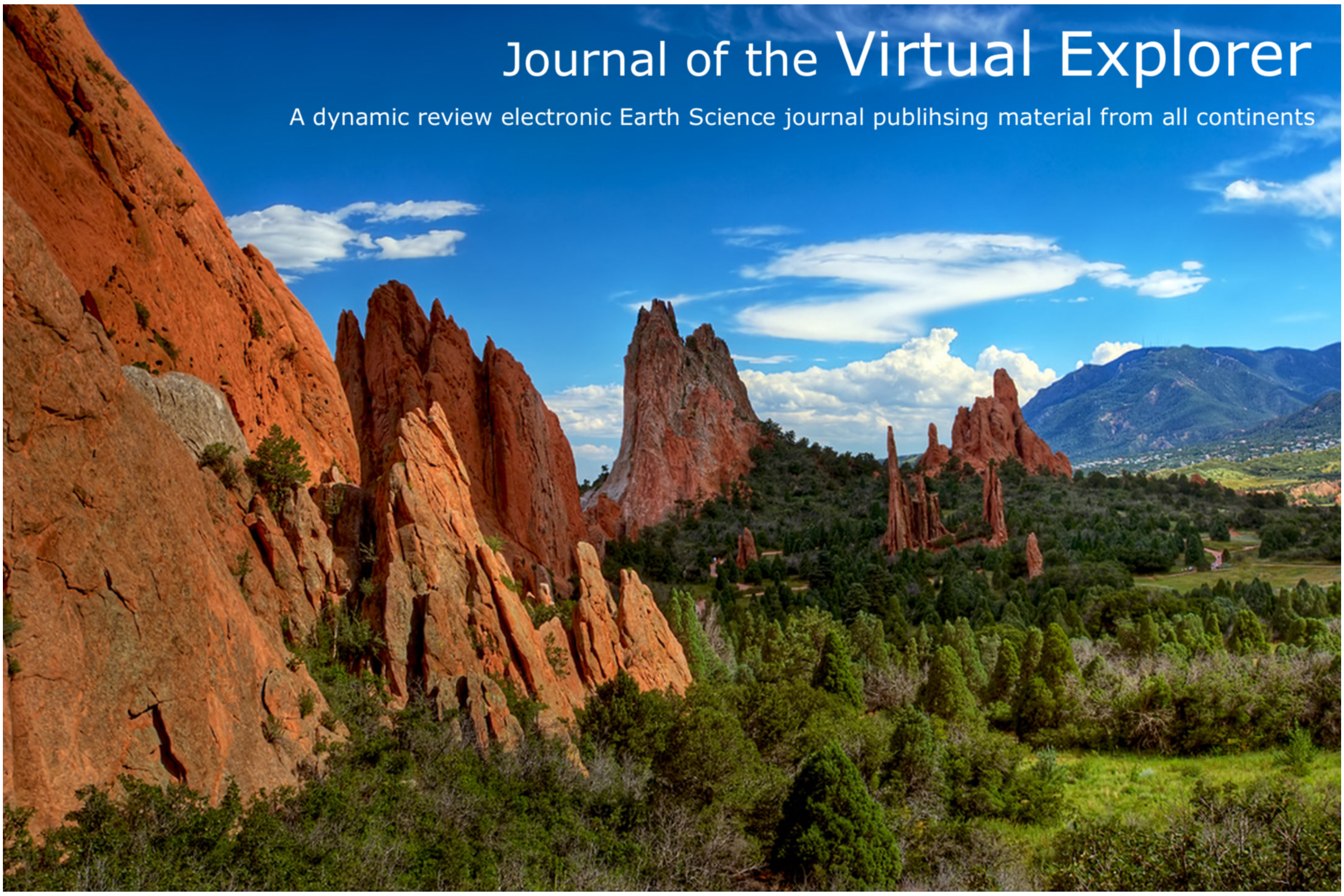

On the Sub-Rotation of a Plate

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Abstract: During their absolute motion over the Earth's surface about their poles of rotation, some plates may experience a further sub-rotation around a second sub-pole. In this case, the instantaneous pole of rotation can be a third separate pole, being the combination of the two basic poles, i.e., the absolute motion-related first pole, and the sub-rotation pole. The sub-rotation pole is the only point of the plate that does not change distance relative to the absolute pole. An analytical method can separate the sub-rotation from the absolute plate motion when sufficient space geodesy data are available.

We applied this model to North America, which is moving WNW-ward in an absolute reference frame, with the first pole located at -64.30°N and 105.52°E; the plate contemporaneously sub-rotates counter-clockwise about an internal pole located at 50.78°N and -77.78°E. The combination of the two poles generates a third migrating instantaneous apparent pole of rotation, that is located at -1.55°N and -82.59°E, which does not comprehensively describe alone the composite motion of the plate.

http://virtualexplorer.com.au/article/2004/86/on-the-sub-rotation-of-a-plate

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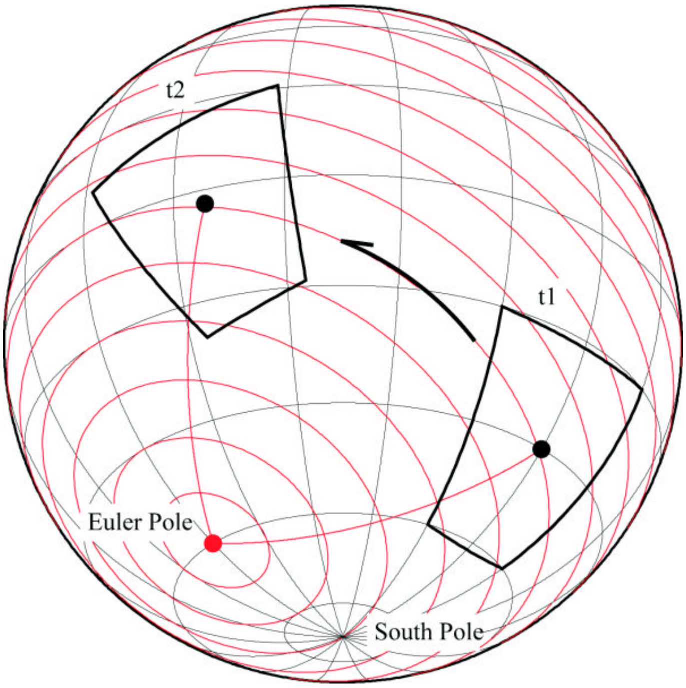
Introduction

Plate motion analysis has been computed in a number of different ways, from magnetic anomalies [Vine 1966], seismology, hotspots [DeMets et al. 1990], [Gripp and Gordon 1990] and space geodesy data [Sella et al. 2002], [Drewes and Meisel 2003]. Plate movements can be described both in absolute reference frames, e.g. the hotspot reference frame [Morgan 1972], [Wilson 1973], or relative reference frames, e.g., one plate relative to another (Chase 1972, Minster and Jordan 1978, DeMets et al. 1990, Gordon 1995). For sake of simplicity, the motion of a plate can be described, in an analytic way, as a rigid body rotation around an Euler pole over the Earth.s surface.

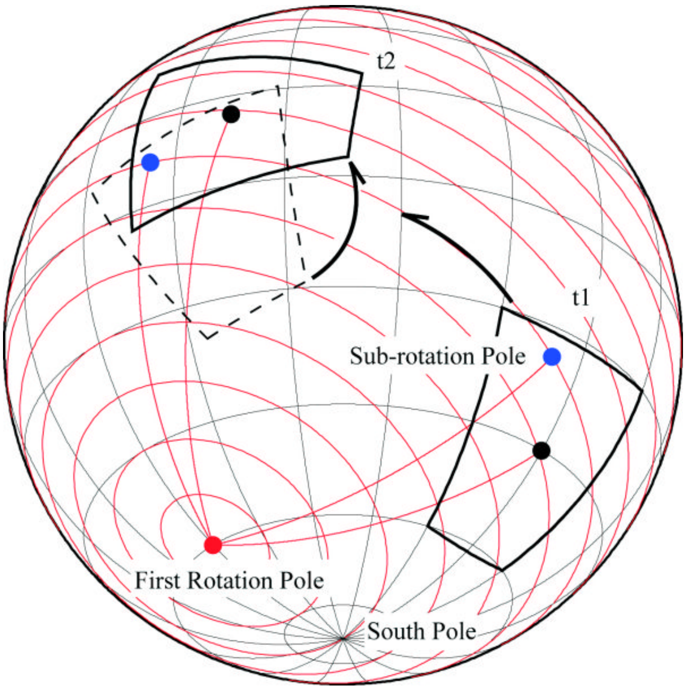
Considering the motion of plates in an absolute reference frame, parts of a plate, while it is rotating about the same Euler pole as the remainder, can make a further contemporaneous rotation with respect to another pole. For example, along with the rifting between Eurasia and North America in the Late Cretaceous and Early Cenozoic, Iberia has rotated counter-clockwise, producing the extension in the Bay of Biscay and most of the shortening in the Pyrenees (Van der Voo 1993). From an absolute motion point of view the 1st-order rotation pole of the Iberia plate was the same of the Eurasia plate, but there was another sub-pole that was contemporaneously involved in the extension of the Bay of Biscay.

Figure 1a. Plate moving on a sphere about an Euler pole.



For the simplest case (a) a plate moves on a sphere constantly rotating about an Euler pole. A point in the plate maintains the same distance from the pole, here shown at two different times, t_1 and t_2.

Figure 1b. Plate moving on a sphere about an Euler pole.



When plate motion is described by two simultaneous motions (b), the plate is rotating about a 1st rotation pole while contemporaneously rotating about a sub-rotation pole. Any reference point in the plate changes its distance from the main pole, shown here for two different times, t_1 and t_2, e.g., increasing or decreasing its distance from the main pole. Only one point does not change its distance from the main pole, and this is the sub-rotation pole that defines the axis of the second rotation.

It is clear that relative kinematic analysis, made for example by the NUVEL-1 relative plate motion model [DeMets et al. 1990] does not allow the hypothesis of sub-rotation, to be verified, because the motion of rigid plates is described, and only one rotation is necessary for plate kinematics. When the relative motion of one plate respect to a second plate is computed, the relative rotation of sub-plates is taken into account either by averaging, or it is ignored. Plate motion models relative to hot spots, such as HS2-NUVEL1 or HS3-NUVEL1A [Gripp and Gordon 1990, 2002] or with the no-net-rotation condition, as NNR-NUVEL1 (Argus and Gordon 1991) also do not recognize the sub-rotation of some plates, because again these are an adaptation of the relative plate motion model NUVEL-1 [DeMets et al. 1990].

For the case of Iberia, in fact, its rotation can be studied relative to Eurasia considered as fixed, and only one angular velocity and one rotation pole is sufficient to describe the motion. But the Eurasia plate was rifted from North America and the Iberia plate has also felt the effect of this motion, and it has followed the Eurasia trajectory. Incremental motion can always be described by a single rotation pole and a rotation. Angular velocity can similarly be described by a rotation pole and an angular velocity. But if we seek an expression that does not change with every instant, two angular velocities and two rotation poles are required to describe the Iberia plate motion.

The previous example focuses attention on plate motion over geological time, where to determine the rotation of a plate and sub-rotation of its elements, paleomagnetism and plate reconstructions have to be involved. We can also consider present-day plate motions in this context, seeking a decomposition of present day motions into elements that involve rotation poles fixed with respect to material coordinates. The NNR reference frame, NNR-NUVEL1 (Argus and Gordon 1991) is based on geological and geophysical data acquired on plate boundaries. Actual plate motion models, REVEL2000 (Sella et al. 2002) or APKIM2002 (Drewes and Meisel 2003) are based on the space geodesy techniques. In these cases also the instantaneous rotation of a plate can be viewed as composed of a primary rotation and a sub-rotation (if required). This decomposition can be performed if the instantaneous pole of rotation will change its material location with time. Under such circumstances points on plates do not move on circular trajectories, but they describe curves that resemble parts of epicycloids projected on the Earth's surface (Figure 2). In reality a plate the plate points chosen would describe only segments of these epicycloids, because insufficiently large rotations would accumulate on the second pole.

Figure 2. Plate motion described by two rotation poles.

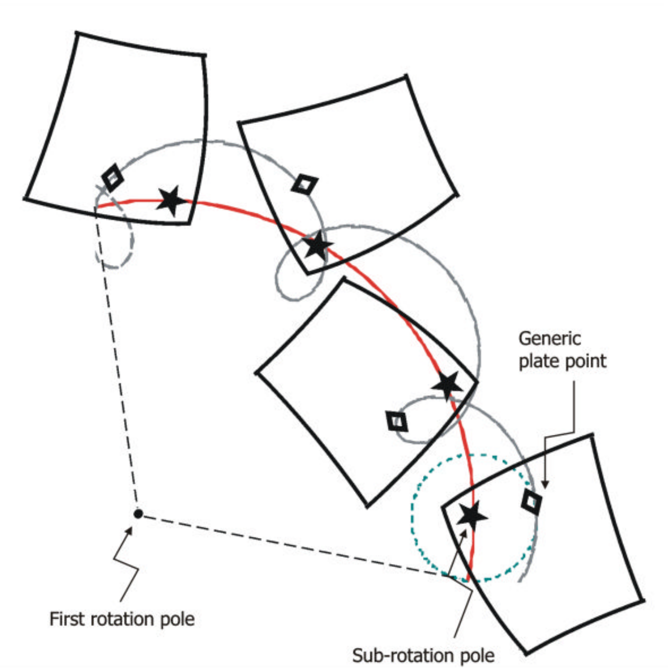


Plate motion when two rotation poles and two angular velocities are required. The plate rotates about the 1st rotation pole while it contemporaneously rotates around the sub-rotation pole. A random point in the plate changes its distance from the main pole at (here) shown at four different times and positions, making a trajectory like an epicycloid (gray line). Only one point does not change its distance with the main pole, and that point becomes the sub-rotation pole. This will make a circular trajectory (red line) around the main pole. For sake of simplicity the two-rotation motion is shown here for a plane, but the decmposition is also valid on a spherical surface. This plate is moving CCW about the 1st rotation pole, at the same time it is sub-rotating CCW about the sub-rotation pole. In reality, the motion of a plate would allow definition of only part of this epicycloidal trajectory.

The importance of detecting sub-rotation in absolute reference frames concerns the dynamical implications, and hypotheses about global plate tectonics. Regardless of the forces that move plates, in general sub-rotation of a sub-plate requires the action of a torque, as in the example of Iberia.

In this paper we describe an analytical method that decomposes the motion of a single plate in an absolute reference frame into a combination of two contemporaneous rotations: a primary rotation around a main pole of rotation (1st rotation pole) and a secondary rotation (sub-rotation) around a second pole. To illustrate the method we used present-day plate motions from the NASA database (Heuin et al. 2004) and the actual plate motion model APKIM2002 (Drewes and Meisel 2003) applied to the North American plate. In the ITRF2000 reference frame the instantaneous motion of the North American Plate (NA) can be interpreted as a combination of two contemporaneous rotation: we can view its motion as the combination of a westward drift, but with a further internal CCW rotation.

Theory and method

Though the application of Euler.s theorem for plate kinematics (Bullard et al. 1965) is always valid, independently of the choice of relative or absolute reference frame, plate motion is principally using relative kinematic analysis, (Cox and Hart 1986, Fowler 1990, Turcotte and Schubert 2001). Present-day motion of a single plate can be described as an instantaneous rigid body rotation on a spherical surface about an Euler pole (Figure 1a). In the simplest case, as time goes on, the plate has a fixed pole of rotation, and any point in the plate maintains the same distance from the pole while the plate moves. Then each point on the plate simply rotates around the pole, travelling on a circular trajectory.

Here, we propose an analytical method that describes this simple motion of one plate point, giving, as the result, the velocities of the transversal and longitudinal displacement depending only by the point coordinates. Because the plate is rigid, its kinematics can be studied starting from geometrical considerations, obtaining motion equations without any dynamical treatment.

Considering the Earth as a sphere, spherical trigonometry (Ayres 1954, Butler 1992, Fowler 1990, Turcotte and Schubert 2001) allows derivation of two equations that describe respectively the velocity of the transversal and of the longitudinal displacement of a single plate point (see appendix A) as follows:

$$V^{(\theta)} = \mathrm{R}\omega \left[\sin\left(\lambda(t) - \Lambda^{(E)}\right) \cos\Theta^{(E)} \right]$$

$$V^{(\lambda)} = \mathrm{R}\omega \cos\theta(t) \left[\sin\Theta^{(E)} - \tan\theta(t)\cos\Theta^{(E)}\cos\left(\lambda(t) - \Lambda^{(E)}\right) \right]$$

where $(\Theta^{(E)}, \Lambda^{(E)})$ are respectively the latitude and the longitude of the Euler pole that are time-independent, $(\theta(t),\lambda(t))$ are the coordinates of the generic plate point that is time-dependent, and where ω is the angular velocity. Because the motion is smooth and steady, $V^{(\theta)}$, $V^{(\lambda)}$ and ω are constant. Then, the expressions to obtain the magnitude and the azimuth of the velocity of a geographical plate point are the following:

$$V = \sqrt{\left(V^{(\lambda)}\right)^2 + \left(V^{(\theta)}\right)^2}$$

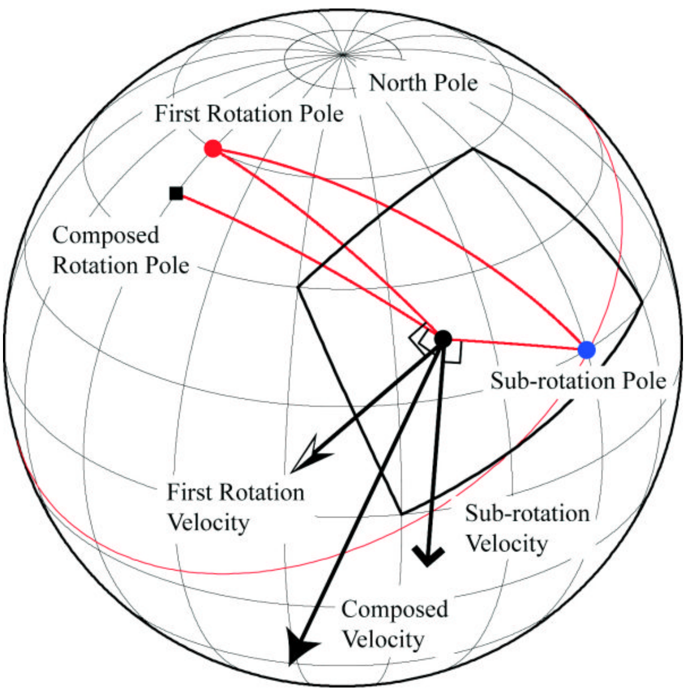
$$\mathrm{azimuth} = \left(\frac{\pi}{2}\right) - \tan^{-1}\left(\frac{V^{(\theta)}}{V^{(\lambda)}}\right)$$

Equation (4) ensures that zero is aligned with the true north to correspond to the geological convention of measuring direction of strike (Henderson 2001).

Equations (1-4) do not differ from the results of Stein and Wysession (Stein and Wysession 2003) and Zhong (Zhong 2001).

In this paper, we consider that a generic lithospheric plate is moving on a spherical surface in an absolute reference frame, making two contemporaneous rotations, one about an external 1st rotation pole, and another about an internal pole (sub-rotation pole), inside the plate that moves with the plate itself (Figure 1b). During the continuum of time, the plate still moves about the 1st rotation pole, but it contemporaneously rotates around the sub-rotation pole, and one reference point of the plate changes its distance from the main pole, shown at two different times t_1 and t_2, increasing and decreasing the distance from the main pole (Figure 1b) and making a particular trajectory like an epicycloid (Figure 2). Only one point does not change its distance with the main pole, and this is the sub-rotation pole. This rotates about the 1st pole and makes a circular trajectory (Figure 2). If the motion is considered without such a decomposition, at every instant there is a different instantaneous pole of rotation, the (composed rotation pole) (Figure 3). A succession of such poles would be necessary to describe the real motion, whereas the two stage decomposition allows the motion to be described simply.

Figure 3. Composed rotation pole



When plate motion is composed by a first rotation and a contemporaneously subrotation, one reference point is characterized by two linear velocities, one relative to the first rotation pole and the other relative to the sub-rotation pole. The sum of these two velocities is the composed velocity related to a third pole of rotation, (composed rotation pole).

Butler (1992) describes the motion of a single plate as a rotation with respect to an internal rotation axis and with respect to an external Euler pole, but he considers two different and separated cases. In the two separated situations only one angular velocity and one Euler pole are required. Using the two rotation model, with one the internal and one external axis, we need two angular velocities, one (ω_r) related to the 1st rotation and the other (ω_s) related to the sub-rotation. Then equations (1) have to be applied twice, the 1st time for the 1st rotation and the second time for the sub-rotation. In this case, using equations (2) at one generic plate point two linear velocities are applied that represent the action of the angular velocities (Figure 3). The resulting velocity is a vector that is instantaneously related to the composed rotation pole that during the continuum of time, changes its position every instant.

Then considering the 1st rotation, we have the following:

$$V_r^{(\theta)} = \mathrm{R}\omega_r \left[\sin\Big(\lambda(t) - \Lambda^*\Big) \cos\Theta^* \right]$$

$$V_r^{(\lambda)} = \mathrm{R}\omega_r \cos\theta(t) \left[\sin\Theta^* - \tan\theta(t)\cos\Theta^* \cos\Big(\lambda(t) - \Lambda^*\Big) \right]$$

where (Θ^*, Λ^*) are respectively the latitude and the longitude of the 1st rotation pole, and these are time-independent. ($\theta(t)$, $\lambda(t)$) are the coordinates of the generic plate point, and these are time-dependent. ω_r is the 1st rotation angular velocity.

Considering also the contemporaneous sub-rotation, equation (1-2) can be written as follows

$$V_s^{(\theta)} = \mathrm{R}\omega_s \left[\sin\Big(\lambda(t) - \Lambda(t)\Big) \cos\Theta(t) \right]$$

$$V_s^{(\lambda)} = \mathrm{R}\omega_s \cos\theta(t) \Big[\sin\Theta(t) - \tan\theta(t)\cos\Theta(t) \cos\Big(\lambda(t) - \Lambda$$

where ($\Theta(t)$, $\Lambda(t)$) are respectively the latitude and the longitude of the sub-rotation pole (time-dependent), ($\theta(t)$, $\lambda(t)$) are the coordinates of the generic plate point (also time-dependent) and ω_r is the sub-rotation angular velocity.

Adding the two contemporaneous rotations, e.g. 1st rotation and sub-rotation, we obtain the expression of the composed movements as follows:

$$V_c^{(\theta)} = V_r^{(\theta)} + V_s^{(\theta)}$$

$$V_c^{(\lambda)} = V_r^{(\lambda)} + V_s^{(\lambda)}$$

where $V_c^{(\theta)}$ and $V_c^{(\lambda)}$ are respectively the velocity of the transversal and longitudinal composed displacement of a single plate point. In this case, the expressions to obtain the magnitude and the azimuth of the velocity of a geographical plate point are the following:

$$V = \sqrt{\Big(V_r^{(\lambda)} + V_s^{(\lambda)}\Big)^2 + \Big(V_r^{(\theta)} + V_s^{(\theta)}\Big)^2}$$

$$\text{azimuth} = \left(\frac{\pi}{2}\right) - \tan^{-1}\left(\frac{V_r^{(\theta)} + V_s^{(\theta)}}{V_r^{(\lambda)} + V_s^{(\lambda)}}\right)$$

and the velocity of the generic plate point obtained with equation (6) is generally related with a third pole of rotation (composed rotation pole), that is not significant in showing the pattern of the overall movement (Figure 3).

For the purpose of separating the two-rotation motion, we need to know the motion parameters as the 1st rotation pole coordinates (T^*, Λ^*) and the 1st rotation angular velocity ω_r and then, the sub-rotation pole coordinates ($\Theta(t)$, $\Lambda(t)$) and the sub-rotation angular velocity ω_s.

If we try to ascertain the two-rotation motion of a plate in an absolute reference frames, e.g. the hotspot frame (Chase 1978, Gripp and Gordon 1990, 2002) or the GPS frame (Heuin et al. 2004) the secondary rotation needs to be separated from the primary motion.

To 2nd the motion parameters, we have to examine tectonic aspects that suggest a two-rotation motion, as indicated in the example of Iberia. Potentially the present day plate motion or the actual plate motion from space geodesy for a single plate can be interpreted as a 1st rotation and a contemporaneous sub-rotation.

Using the GPS velocties solution (Heuin et al. 2004) and the actual plate motion model APKIM2002, (Drewes and Meisel 2003) we tried to apply equations (6) to the North American plate, hypothesizing that the instantaneous North American plate motion can be viewed as the result of the sum of two different rotations, one an external 1st rotation and the other a contemporaneous sub-rotation.

The sub-rotation of the North America and discussion

The application of the equations (5) and (6) to the North American plate is 1stly a test for the analytical exactness of the two-rotation plate motion model. There is no clear geodynamical evidence (as is the case for the example of Iberia) that suggests the North American plate motion is composed by two contemporaneous rotations, though Doglioni (1990) suggests such a sub-rotation of the North American plate can be justified.

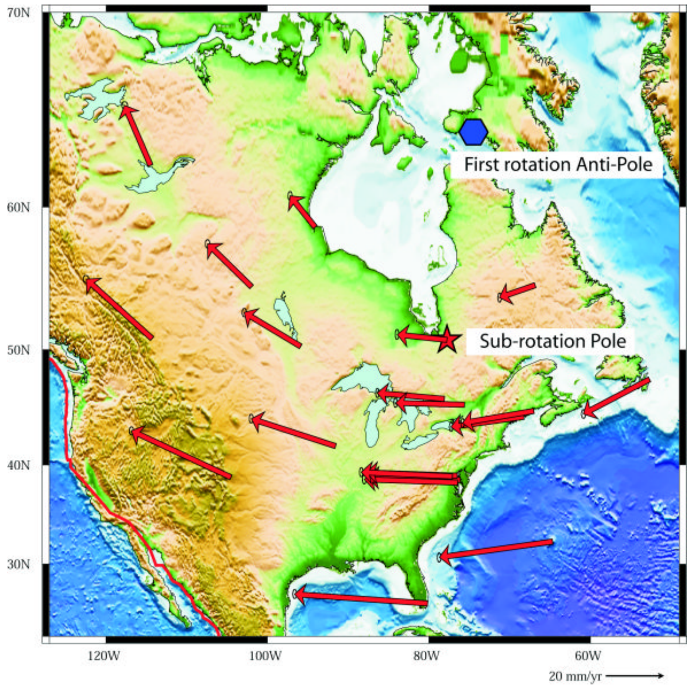
This plate was chosen also because it has the largest number of GPS stations in the global network and their velocities are reported in an absolute reference frame, ITRF2000 (Altamimi et al. 2002), and this is useful for comparison with model results. In this reference frame a no-net-rotation condition (NNR) is conventionally imposed.

Using the angular velocities of the present-day plate kinematic and deformation model (APKIM2002) (Drewes and Meisel 2003), we suppose a priori that the North American 1st rotation pole is the APKIM2002 Pacific rotation pole located at (-64.30 ± 0.18) °N and (105.52 ± 1.15) °E with an angular velocity ω_r = (0.6588 ± 0.0039) °.Myr-1.

To align the two-rotation plate motion to the GPS velocities distribution (Heuin et al. 2004) it is necessary to introduce a contemporaneous sub-rotation on the 1st rotation and the result is a sub-rotation pole located at (50.78 ± 0.06) °N, which is not far from the rotation pole of Stein and Sella (2002) and (-77.78 ± 0.50) °E with a sub-rotation angular velocity ω_s = (0.7594 ± 0.0069) °. MYr^{-1} . With these two sets of motion parameters, i.e., 1st rotation and sub-rotation, we applied equation (3) then (4) for 20 selected intraplate space geodesy stations, far from the diffusely deformed western North America plate margin. The results are shown in Table 1.

Figure 4 and Figure 5 show respectively and separately the 1st rotation of the GPS stations about the 1st rotation pole and their sub-rotation about the sub-rotation pole. When we consider only the sub-rotation, the sub-rotation pole is fixed (Figure 5), but if there is also the 1st rotation, the sub-rotation pole moves along a parallel of the 1st rotation pole (Figure 4).

Figure 4. Sub-rotation of the North America

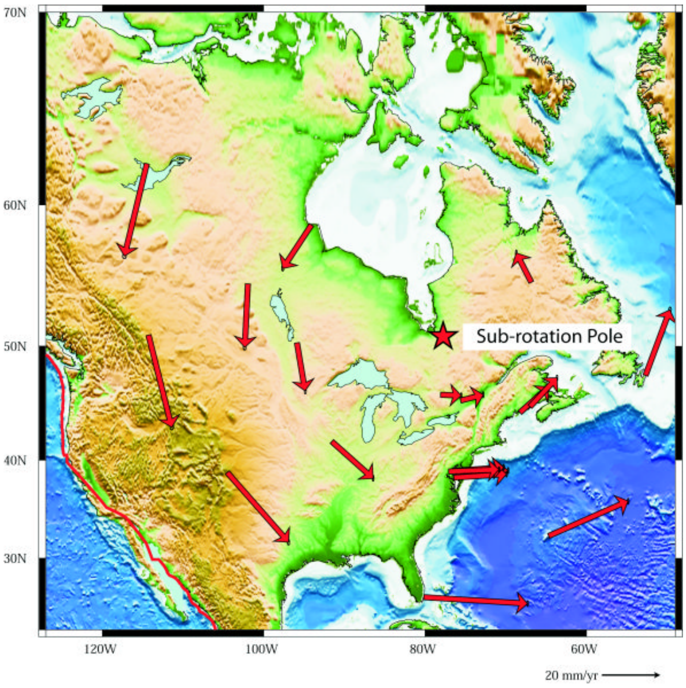


Assuming that the first rotation of the North American plate is along the pacific trend (see text), the selected GPS stations of the North American plate rotate firstly about the

APKIM2002 Pacific Euler pole [Drewes and Meisel, 2003]. Ellipses show the 2-D 95% confidence ellipse of the velocity. Here, we report the Pacific Euler anti-pole located at 64:30°N and -74:48°E with a first rotation angular velocity o_r = (0:6588 ± 0:0039)° Myr^{-1}.

In this case sub-rotation pole is moving with $V_r^{(\Theta)}$ = (1:83 ± 0:70) $mmyr^{-1}$ and $V_r^{(\Theta)}$ = (-17:17 ± 0:25) $mmyr^{-1}$.

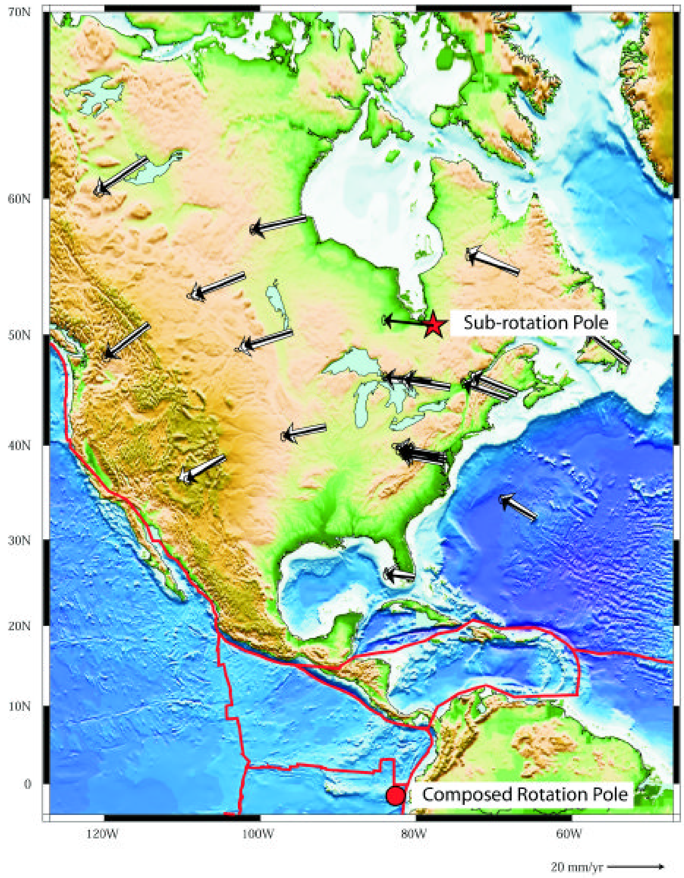
Figure 5. North American plate



If there were only the sub-rotation, the North American plate would rotate about a sub-rotation pole located at (50:78 ± 0:06)°N and (-77:78 ± 0:50)°E with an angular velocity o_s = (0:7594 ± 0:0069)° Myr^{-1}. Ellipses show the 2-D 95% confidence ellipse of the velocity.

For the case of North American plate, the sub-rotation pole is moving with a transversal velocity $V_r^{(\theta)}$ = (1.83 ± 0.70) $mm.yr^{-1}$ and a longitudinal velocity $V_r^{(\lambda)}$ =(-17.17 ± 0.25) mm.yr-1 (Figure 4). Then, we applied equation (5) and equation (6) and the results are shown in Figure 6 and in Table 2.

Figure 6. Adding the two contemporaneous rotations

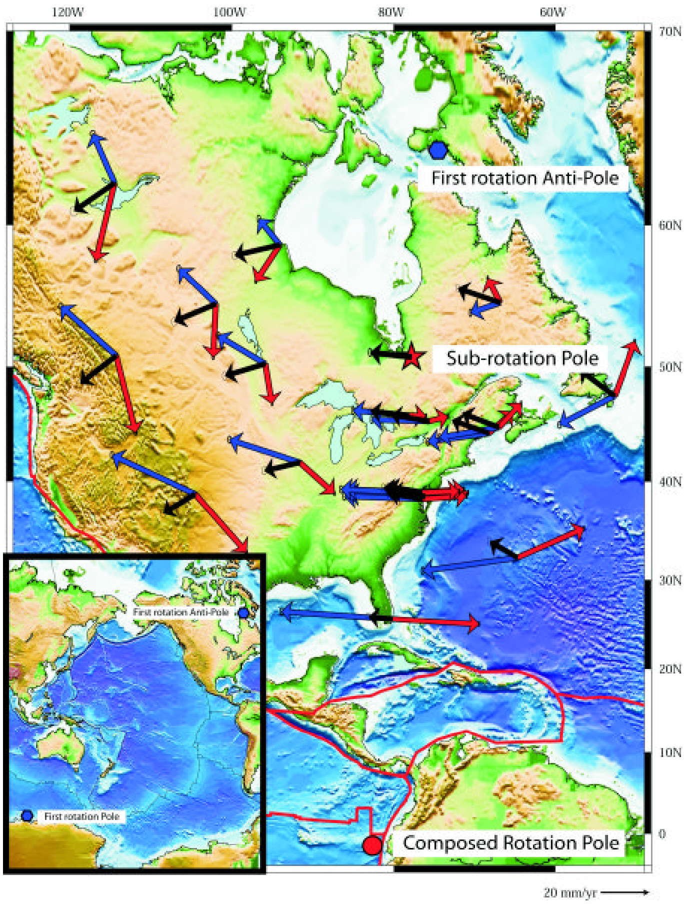


Adding the two contemporaneous rotations, comparison between North American GPS selected station velocities (white vectors) and composed motion model results (black vectors) show good agreement.

They are both related to a composed rotation pole located at (-1:55 ± 0:77)°N and (-82:59 ± 0:35)°E that represents their interplay. Ellipses show the 2-D 95% confidence ellipse of the velocity.

Comparison between GPS data (Heuin et al. 2004) and model results indicate good agreement and their interference generates a third composed rotation pole located at (-1.55 ± 0.77) °N and (-82.59 ± 0.35) °E. In Figure 7, we report a complete picture where the two instantaneous rotations for the plate, i.e., 1st rotation and sub-rotation are shown. It is clear how this sum gives an instantaneous velocity distribution related with an instantaneous third composed pole. The instantaneous rotation represented by the third composed pole would represent the plate motion for only in one instant, and that would be motion on an epycicloid that represented the trajectory for a plate motion with two contemporaneous rotations (Figure 2).

Figure 7. Composed motion for North American plate



Summarizing picture when it is represented the composed motion for North American plate. For the 20 selected GPS station, the first rotation (blue vectors) is about the first rotation pole located at (-64:30 ± 0:18)°N and (105:52±1:15)°E with o_r = (0:6588 ± 0:0039)° Myr^{-1}, the sub-rotation (red vectors) is about the sub-rotation pole located at (50:78 ± 0:06)°N and (-77:78 ± 0:50)°E with an angular velocity o_s = (0:7594 ± 0:0069)&edeg Myr^{-1}. Composition of these two contemporaneous movements (black vectors) are related to a composed rotation pole located at (-1:55 ± 0:77)°N and (-82:59 ± 0:35)°E that is not significant of the composed motion. Ellipses show the 2-D 95% confidence ellipse of the velocity.

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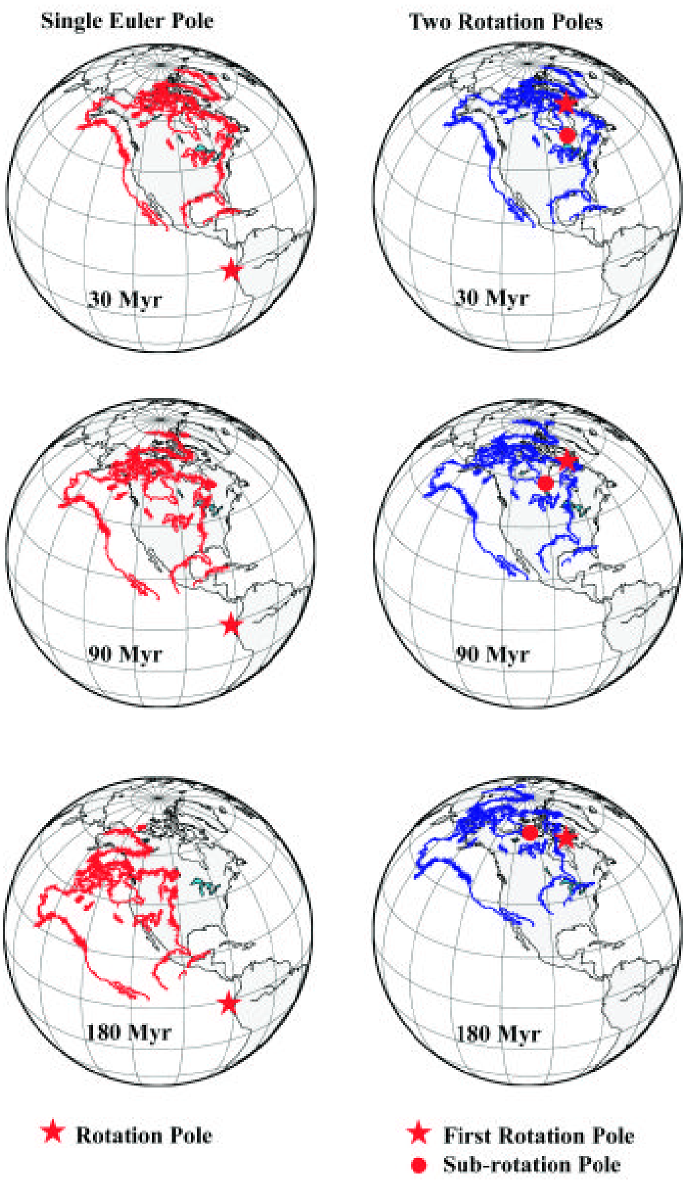
A. The equations of motion

During plate motions, choosing the North Pole (O) as the origin of the coordinate system, in the spherical triangle OEQ (Figure 8), the length of the arc a = EQ, is an invariant relative to the time. In a sphere of a unit radius, from spherical trigonometry rules (Ayres 1954), we have:

Figure 8. Simple plate motion



For the simple plate motion, in the spherical triangle OEQ, Q is the generic plate point, E is the Euler pole, and the length a = EQ is an invariant during plate motion.

$$\cos\left(\frac{\pi}{2}-\theta(t)\right)=\cos a\cos\left(\frac{\pi}{2}-\Theta^{(E)}\right)+\sin a\sin\left(\frac{\pi}{2}-\Theta^{(E)}\right)\cos\beta(t)$$

$$\sin a=\frac{\sin\left(\lambda(t)-\Lambda^{(E)}\right)\sin\left(\frac{\pi}{2}-\theta(t)\right)}{\sin\beta(t)}$$

and differentiating the (A1) relative to the time, using the (A2) and simplifying, we obtain:

$$\frac{\mathrm{d}}{\mathrm{d}t}\theta(t)=\sin\left(\lambda(t)-\Lambda^{(E)}\right)\cos\Theta^{(E)}\frac{\mathrm{d}}{\mathrm{d}t}\beta(t)$$

Now, considering the Earth as a sphere with a radius R=6371 km, being $V^{(\theta)}$ = R.(δ/δt)θ(t) and ω_r = (δ/δt) β(t), we obtain the velocity of the transversal displacement as equation (1).

To calculate, instead, the velocity of the longitudinal displacement, we consider the following:

$$\cos a=\cos\left(\frac{\pi}{2}-\theta(t)\right)\cos\left(\frac{\pi}{2}-\Theta^{(E)}\right)+\sin\left(\frac{\pi}{2}-\theta(t)\right)\sin\left(\frac{\pi}{2}\right.$$

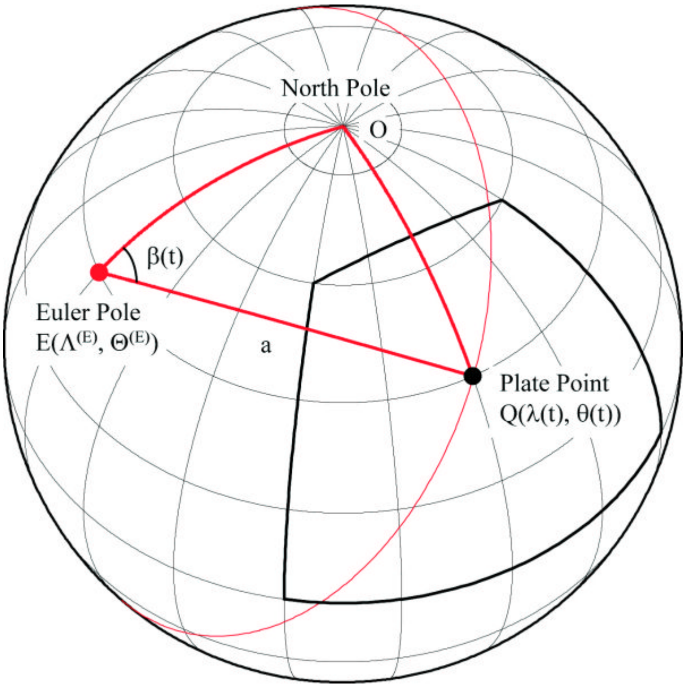
and differentiating the (A4) relative to time, using (A3) and simplifying, we obtain:

$$\frac{\mathrm{d}}{\mathrm{d}t}\lambda(t)=\left[\sin\Theta^{(E)}-\tan\theta(t)\cos\Theta^{(E)}\cos\left(\lambda(t)-\Lambda^{(E)}\right)\right]\frac{\mathrm{d}}{\mathrm{d}t}\beta$$

Then, considering the Earth as a sphere with a radius R=6371 km, being $V^{(\lambda)}$ = R.cos θ(t)(δ/δt)θ(t) and ω= (δ/δt)β(t), we obtain equation (2).

When we separately consider the two components of the composed motions, e.g., the 1st rotation and the sub-rotation, there are two spherical triangles, (OP^*Q and OPQ) (Figure 9), and for simple motion, the lengths b^*=P^*Q and b = PQ are invariant. For this reason we can calculate the velocities of transversal and longitudinal displacement, respectively, for the 1st rotation and the sub-rotation (equations 5-6), with the same procedure as for the simple case, but writing ω_r = (δ/δt)α^*(t) and ω_s = (δ/δt)α(t). When we add the two components in equation (5), the length b^* =P^*Q is not invariant with time and one reference plate of the plate changes its distance from the 1st rotation pole during motion (Figure 1).

Figure 9. First rotation and the sub-rotation



When the motion is composed by a first rotation and the sub-rotation, there are two spherical triangles, OP*Q and OPQ, and the lengths b* = P*Q and b = PQ are separately invariant. Applying equation (5) the length b* = P*Q is not invariant during the motion. The only point that moves along the parallel of the first rotation pole is the sub-rotation pole.

B. Tables

When applying equation (3) and equation (4) to the 20 GPS stations, selected far from the western US margin, we obtain velocity of transversal and longitudinal displacement ($V_r^{(\lambda)}$, ($V_r^{(\Theta)}$ and ($V_s^{(\lambda)}$, ($V_s^{(\Theta)}$) with their 1σ standard deviation, respectively for the first rotation with a first rotation pole located at (-64:30 ± 0:18)°N and (105:52 ± 1:15)°E and or = (0:6588 ± 0:0039) ° Myr^1 and a sub-rotation pole located at (50:78 ± 0:06)°N and (-77:78 ± 0:50)°E with o_s = (0:7594 ± 0:0069) ° Myr^{-1}

Table B.1. Applying equation (3) and equation (4) to the 20 GPS stations

			First Rotation		**Sub-rotation**	
GPS Station	**Longitude (λ)**	**Latitude (Θ)**	$\mathbf{V^{\lambda}_{GPS} \pm 1\sigma}$	$\mathbf{V^{\Theta}_{GPS} \pm 1\sigma}$	$\mathbf{V^{\lambda}_{GPS} \pm 1\sigma}$	$\mathbf{V^{\Theta}_{GPS} \pm 1\sigma}$
	°E	**°N**	**(mm yr^{-1})**	**(mm yr^{-1})**	**(mm yr^{-1})**	**(mm yr^{-1})**
ALGO	-78.07	45.96	-23.10 ± 0.23	1.99 ± 0.64	7.71 ± 0.09	-0.28 ± 0.41
AMC2	-104.52	38.80	-34.21 ± 0.31	15.91 ± 0.57	21.09 ± 0.16	-24.02 ± 0.38
AOLM	-80.16	25.73	-45.74 ± 0.27	3.15 ± 0.64	35.75 ± 0.09	-2.22 ± 0.41
BARH	-68.22	44.39	-25.07 ± 0.24	-3.46 ± 0.63	9.91 ± 0.11	8.86 ± 0.41
BRMU	-64.69	32.37	-38.99 ± 0.27	-5.40 ± 0.63	27.40 ± 0.11	12.08 ± 0.40
CHUR	-94.08	58.75	-8.65 ± 0.29	10.66 ± 0.61	-9.88 ± 0.15	-14.99 ± 0.40
DUBO	-95.86	50.25	-19.46 ± 0.29	11.58 ± 0.60	2.80 ± 0.14	-16.57 ± 0.40
EPRT	-66.99	44.90	-24.51 ± 0.25	-4.14 ± 0.63	9.30 ± 0.11	9.98 ± 0.40
FLIN	-101.97	54.72	-15.11 ± 0.31	14.67 ± 0.58	-1.98 ± 0.18	-21.88 ± 0.38
HNPT	-76.13	38.58	-31.79 ± 0.25	0.92 ± 0.64	17.84 ± 0.09	1.53 ± 0.41
NLIB	-91.57	41.77	-29,00 ± 0.27	9.34 ± 0.61	14.24 ± 0.12	-12.73 ± 0.40
NRC1	-75.62	45.45	-23.67 ± 0.24	0.63 ± 0.64	7.86 ± 0.09	2.00 ± 0.41
NRC2	-75.62	45.45	-23.67 ± 0.24	0.63 ± 0.64	7.86 ± 0.09	2.00 ± 0.41
PRDS	-114.29	50.87	-22.73 ± 0.38	20.34 ± 0.52	7.99 ± 0.23	-31.76 ± 0.35
SCH2	-66.83	54.83	-12.28 ± 0.24	-4.23 ± 0.63	-5.17 ± 0.12	10.13 ± 0.40
SOL1	-76.45	38.31	-32.10 ± 0.25	1.09 ± 0.64	18.21 ± 0.09	1.23 ± 0.41
STJO	-52.67	47.59	-22.73 ± 0.29	-11.80 ± 0.60	8.41 ± 0.17	22.63 ± 0.38
USNA	-76.47	38.98	-31.34 ± 0.25	1.11 ± 0.64	17.26 ± 0.09	1.20 ± 0.41
USNO	-77.06	38.91	-31.42 ± 0.25	1.43 ± 0.64	17.35 ± 0.09	0.66 ± 0.41
YELL	-114.48	62.48	-8.92 ± 0.41	20.42 ± 0.51	-7.73 ± 0.26	-31.90 ± 0.35

Comparison between GPS data [Heflin et al., 2004] and model results indicates good agreement. GPS and model longitudinal and transversal velocity components (...) are reported with their 1σ standard deviation. Results were calculated studying stable NA plate regions and choosing GPS stations far from western U.S. diffused plate margin. Equations were applied with a sub-rotation pole located at (50:78 ± 0:06)°N and (-77:78 ± 0:50)°E and with a sub-rotation angular velocity o_s = (0:7594 ± 0:0069) ° Myr^{-1} and a first angular velocity or = (0:6588 ± 0:0039) ° Myr^{-1} applied on the first rotation pole located at (-64:30 ± 0:18)°N and (105:52 ± 1:15)°.

Table B.2. Comparison between GPS data

			GPS Data		Composed Movement	
GPS Station	**Longitude (λ)**	**Latitude (Θ)**	$V^{\lambda}_{GPS} \pm 1\sigma$	$V^{\Theta}_{GPS} \pm 1\sigma$	$V^{\lambda}_{GPS} \pm 1\sigma$	$V^{\Theta}_{GPS} \pm 1\sigma$
	°E	**°N**	**(mm yr^{-1})**	**(mm yr^{-1})**	**(mm yr^{-1})**	**(mm yr^{-1})**
ALGO	-78.07	45.96	-16.82 ± 0.07	1.17 ± 0.04	-16.00 ± 0.34	1.71 ± 0.65
AMC2	-104.52	38.80	-16.25 ± 0.15	-7.84 ± 0.23	-13.12 ± 0.47	-8.11 ± 0.69
AOLM	-80.16	25.73	-9.99 ± 0.08	2.29 ± 0.16	-9.99 ± 0.36	0.92 ± 0.68
BARH	-68.22	44.39	-15.56 ± 0.17	6.11 ± 0.27	-15.17 ± 0.35	5.39 ± 0.65
BRMU	-64.69	32.37	-11.89 ± 0.02	7.77 ± 0.04	-11.60 ± 0.37	6.68 ± 0.67
CHUR	-94.08	58.75	-18.79 ± 0.06	-4.59 ± 0.10	-18.53 ± 0.43	-4.33 ± 0.69
DUBO	-95.86	50.25	-18.91 ± 0.33	-6.22 ± 0.20	-16.66 ± 0.43	-4.99 ± 0.69
EPRT	-66.99	44.90	-16.22 ± 0.17	6.61 ± 0.27	-15.21 ± 0.36	5.86 ± 0.65
FLIN	-101.97	54.72	-18.91 ± 0.16	-7.41 ± 0.10	-17.10 ± 0.51	-7.21 ± 0.71
HNPT	-76.13	38.58	-17.25 ± 0.32	4.64 ± 0.22	-13.96 ± 0.34	2.45 ± 0.66
NLIB	-91.57	41.77	-14.41 ± 0.03	-2.96 ± 0.04	-14.76 ± 0.40	-3.39 ± 0.67
NRC1	-75.62	45.45	-15.97 ± 0.07	2.96 ± 0.10	-15.80 ± 0.34	2.64 ± 0.65
NRC2	-75.62	45.45	-17.27 ± 0.19	2.81 ± 0.28	-15.80 ± 0.34	2.64 ± 0.65
PRDS	-114.29	50.87	-14.98 ± 0.08	-11.34 ± 0.12	-14.73 ± 0.61	-11.41 ± 0.74
SCH2	-66.83	54.83	-17.70 ± 0.09	7.95 ± 0.12	-17.45 ± 0.36	5.90 ± 0.65
SOL1	-76.45	38.31	-14.86 ± 0.04	2.72 ± 0.08	-13.88 ± 0.34	2.33 ± 0.66
STJO	-52.67	47.59	-14.91 ± 0.10	11.88 ± 0.08	-14.32 ± 0.46	10.84 ± 0.67
USNA	-76.47	38.98	-14.18 ± 0.04	0.63 ± 0.08	-14.07 ± 0.34	2.36 ± 0.66
USNO	-77.06	38.91	-14.40 ± 0.06	3.00 ± 0.10	-14.07 ± 0.34	2.09 ± 0.66
YELL	-114.48	62.48	-17.82 ± 0.11	-11.80 ± 0.08	-16.65 ± 0.67	-11.48 ± 0.76