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Strain Machine
A Macromedia Director 8.5 Application for Introducing the Concepts of Simple and Pure Shear

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Abstract: Strain Machine, a Macromedia Director 8.5 program, and 9 interactive Flash 5 animations were developed to allow students a simple means of discovery through experimentation. Specifically, Strain Machine allows students to conduct simple and pure shear experiments while the 9 interactive Flash 5 animations emphasize the dynamics of the various components of experiments conducted with Strain Machine, and other geometrical and mathematical aspects of deformation and strain.

The mathematical background necessary for students to use Strain Machine is provided as is an exercise that might be fruitfully applied in an introductory structural geology class. Executable versions of Strain Machine and the 9 interactive Flash animations used in this paper are freely downloadable at the author's web site.

Keywords: structure, strain, pure shear, simple shear, plane strain, animations

Introduction

The distortion of the Earth's crust is a dynamic process that occurs in a wide variety of tectonic environments. To completely understand how such distortions arise, as for example, from an initially isotropic granitoid or from a series of horizontal layers, undergraduate students need to first tackle the basic concepts of strain.

Strains accumulate in rocks as a result of the progressive displacement of particles. Such displacements are difficult to visualize through static illustrations like those encountered in textbooks. In an attempt to mitigate this difficulty, as well as other problems commonly encountered in teaching the principles of strain, a cross-platform application called Strain Machine as well as several other interactive animations were developed in Macromedia Director 8.5 and Flash 5. Executable versions of Strain Machine for both the PC and Macintosh are free to download at the author's web site as are executables of all Flash 5 movies used in this paper. Executable versions of Strain Machine allow students to print out the results of their experiments while the Shockwave (web-based) version accompanying this paper does not.

This paper was written for the undergraduate student and professional struggling with the concept of strain. In order to accommodate such readers I first briefly provide basic background information about strain, and then derive the two-dimensional strain transformation equations used in the development of algorithms implemented in Strain Machine. A brief description of the three steps taken to conduct an experiment with Strain Machine is then provided. Finally, I conclude with an exercise that focuses on learning the principles of simple and pure shear through the act of discovery and experimentation with Strain Machine.

Background

Deformation and strain are two terms that are used often in structural geology. Deformation of a rock body may involve a rotation, a translation, a distortion or strain, and a volume change or dilation (Twiss and Moores, 1992; Davis and Reynolds, 1996; Hatcher, 1996; van der Pluijm and Marshak, 1997). As you explore these relationships in Figure 1, note that strain is only synonymous with deformation if there has not been any volume change, translation, or rotation (e.g., van der Pluijm and Marshak, 1997).

A Flash 5 animation of the 3D distortion of a crinoid column is provided in Figure 2. As you observe the distorting crinoid note that its cross-sectional area changes from that of a circle to that of an ellipse? But exactly how did this change take place? In other words, how were the particles along the surface of the crinoid displaced to yield the elliptical form? In order to address this question, we need to establish prior to distortion a set of reference points or lines on the object of interest so that we can measure how such features have changed after deformation has run its course (e.g., Ramsay and Huber, 1983).

Any geologist who has drawn a 3D mesh object in a modern day graphics program such as *Carrara* understands the idea that any imaginary or real object can be thought of as a series of points connected by lines. Utilizing this concept we then can view the end points of the ridges emanating from the center of a crinoid column as a set of material points. Material lines then are imaginary lines connecting any two material points in a rock body or fossil (e.g., lines R and R' in Figure 2) (van der Pluijm and Marshak, 1997).

Homogenous strain occurs when material lines that were parallel before a distortion remain parallel after

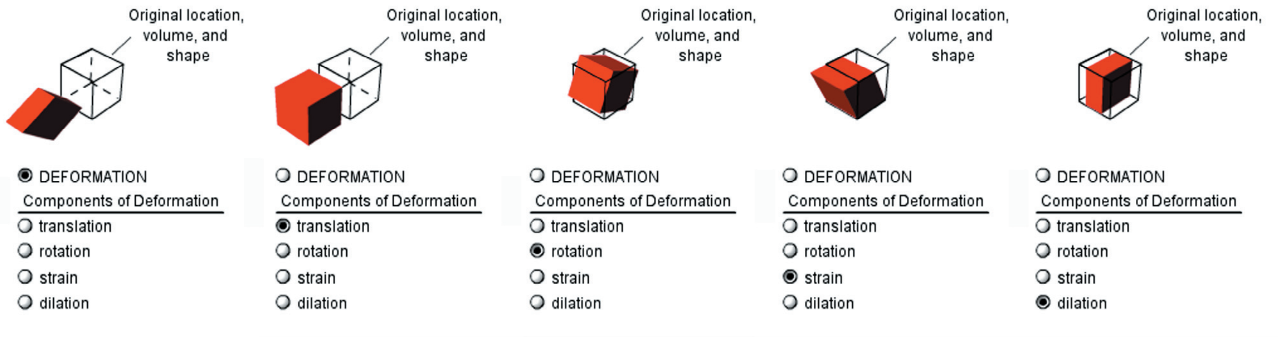

Figure 1. The four components of deformation. Select an item and the click on the Play button. (**Select image for interactive HTML version**)

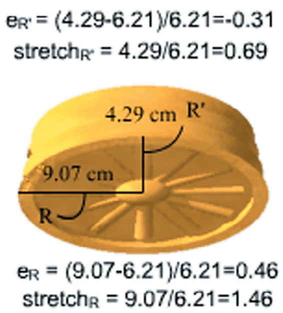

Figure 2. To distort the crinoid column by plane strain pure shear click on the Distort button. R and R' are two material lines that are shortened and lengthened during the distortion. (**Select image for interactive HTML version**)

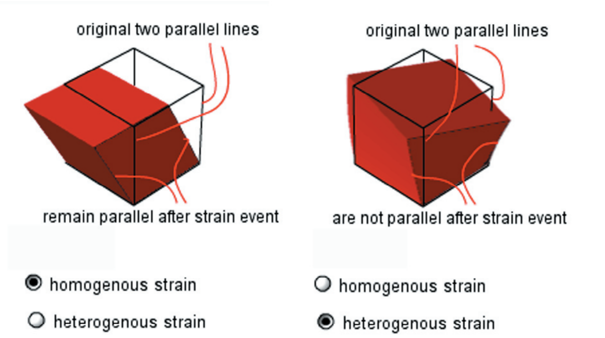

Figure 3. Select a type of strain and then click on the Play button. (**Select image for interactive HTML version**)

the distortion (Fig. 3). A corollary of this statement is that a circle will be transformed into an ellipse during a homogeneous strain event (e.g., Fig. 2). If either of these two statements are not true, then strain is heterogeneous (van der Pluijm and Marshak, 1997). But how do we measure changes in the lengths and orientations of material lines? Below I discuss three parameters that are useful for such characterizations. Others can be found in Means (1976), Ramsay and Huber (1983), Ragan (1985), Twiss and Moores (1992), Hatcher (1996), Reynolds and Davis (1996), and van der Pluijm and Marshak (1997).

Elongation (*e*), a dimension-less quantity, is expressed as

$$e = \frac{\delta l}{l_o} = \frac{(l - l_o)}{l_o} \qquad (1)$$

where l is the strained length, and l_o is the original length of some measurable material line. For example, if the material line labeled R in Figure 2 were increased during a distortion from 6.21 to 9.07 cm, then the elongation would be:

$$e = \frac{(l - l_o)}{l_o} = \frac{(9.07cm - 6.21cm)}{6.21cm} = 0.46$$

In other words, the length of line R was increased by an amount equal to 46% of its original value. Positive values of elongation represent an increase in length while negative values represent a decrease.

Stretch (*s*) is the ratio between (l) and ($l0$). In the symbolic language of mathematics it is

$$s = \frac{l}{l_o} \qquad (2)$$

where, l and l_o are as defined in the previous paragraph. Stretch also can be expressed as

$$s = e + 1 \qquad (3)$$

Returning for a moment to the distortion of line R in Figure 2,

$$s = \frac{l}{l_o} = \frac{9.07cm}{6.21cm} = 1.46$$

Thus, stretch tells us that the length of the distorted line is 146% of its original length. Values of stretch greater than 1.0 represent elongations while values less than 1.0 represent shortening.

Shear strain (γ, gamma), is expressed mathematically as

$$\gamma = \tan \psi \qquad (4)$$

where the angular shear strain (ψ, psi), is the change in angle of two initially perpendicular lines (Fig. 4).

In Figure 4, following the imposed distortion the change in angle between the initially vertical and horizontal lines is 24°; hence,

$$\gamma = \tan(24°) = 0.445$$

As ψ increases from 0 to 90, *tan (ψ)* increases from 0 to infinity. In other words, large values of γ reflect greater amounts of shear strain and rotation than do smaller ones.

The Strain Transformation Equations

Distortion is a non-rigid body operation that produces a change in the shape of a body with or without a change in its volume. Plane strain is a specific type of distortion involving no volume change; hence, it is isochoric (Ragan, 1985). Simple and pure shear represent two end member types of plane strain (Twiss and Moores, 1992; Ferguson, 1994; Davis and Reynolds, 1996; Hatcher, 1996; van der Pluijm and Marshak, 1997).

Simple Shear

In simple shear the displacements of particles are constrained to lie within a plane of shear (Twiss and Moores, 1992; Ferguson, 1994; Davis and Reynolds, 1996; Hatcher, 1996; van der Pluijm and Marshak, 1997). For example, consider some point *a(x, y)* lying within a domain of potential simple shear with a coordinate system like that shown in Figure 5. If the plane of shear is parallel to the x-axis, then after an increment of simple shear point a will not move parallel to the y-axis, but will be translated parallel to the x-axis to some new position *a'(x', y')* (Fig. 5). The distance *v* is the amount that point *a* is translated parallel to the y-axis, while *u* is the distance it is translated parallel to the x-axis (Fig. 5). Vector *h* then is the resultant of these two components. From Figure 5 it follows that

$$v = h * \cos(\psi) \qquad (5)$$

Rearranging and solving for *h* yields

$$h = v / \cos(\psi) \qquad (6)$$

Therefore

$$u = [v / \cos(\psi)] * \sin(\psi) \qquad (7)$$

or upon simplifying

$$u = v * \tan(\psi) \qquad (8)$$

Recall that the shear strain, γ, is tan(ψ), and that under the conditions of simple shear, no displacement parallel to the y-axis is permitted; hence, *v* = *y*, and the distance from x to x' is u = γ**y* (Fig. 5). Putting these relationships in terms of coordinate transformations we then have

$$x' = (1 * x) + (\gamma * y) \qquad (9)$$

and

$$y' = (0 * x) + (1 * y) \qquad (10)$$

Expressing these equations in matrix form we write

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & \gamma \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} \qquad (11)$$

Pure Shear

Pure shear describes the change of points in a body relative to each other without the constraint that particle displacements can only occur parallel to some shear plane. For example, consider some point ***a****(x, y)* lying within a domain of potential pure shear with a coordinate system like that shown in Figure 6. After an increment of strain this point will be translated to some new position ***a'****(x', y')*.

In Figure 6, x_{def} and x_{undef} are the horizontal distances from the y-axis to points **a'** and **a** respectively. The ratio x_{def}/x_{undef} is the fractional change in position parallel to the x-axis that **a'** has undergone relative to its original position (cf., Ramsay and Huber, 1983). As discussed earlier, this ratio is simply the stretch in the x-direction, i.e., *(e + 1)* (Fig. 6). Thus, the coordinate transformation equation for *x'* is

$$x' = [(e+1) * x] + (0 * y) \qquad (12)$$

Now, recall that pure shear is a constant volume process. This constraint implies that the area occupied by the brown rectangle in Figure 6 has to be the same as the area of the blue rectangular region, which represents its deformed equivalent.

Mathematically, we then write

$$\text{Area}_{\text{brown}} = \text{Area}_{\text{blue}} \qquad (13)$$

The right hand side of equation (13) is equivalent to $x_{def} * y_{def}$ (Fig. 6). Substituting this information into equation (13) and rearranging yields

$$\text{Area}_{\text{blue}} / x_{def} = y_{def} \qquad (14)$$

However, Areablue also equals $x_{undef} * y_{undef}$, and we therefore write

$$(x_{undef} * y_{undef}) / x_{def} = y_{def} \qquad (15)$$

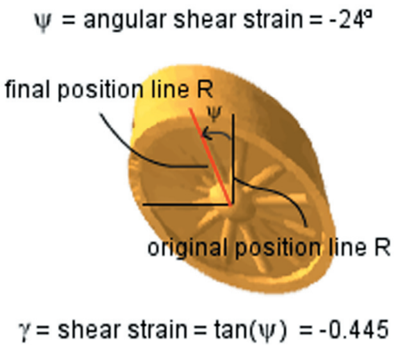

Figure 4. To distort the crinoid column by plane strain simple shear click on the Distort button. R and R' are two material lines that prior to progressive simple shear were perpendicular. Following distortion, the change in angle between R and R' is 24°.(**Select image for interactive HTML version**)

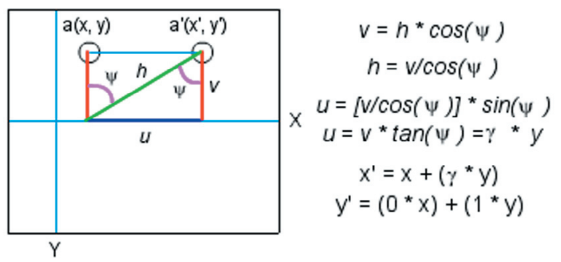

Figure 5. A Flash 5 animation illustrating the key elements in the derivation of the transformation equation for two dimensional simple shear. Select sequentially the items in the Show Geometrical Element drop-down list. To see the equations for the geometrical elements identified in the resulting animation, select the item from the Show Equation For drop-down list. (**Select image for interactive HTML version**)

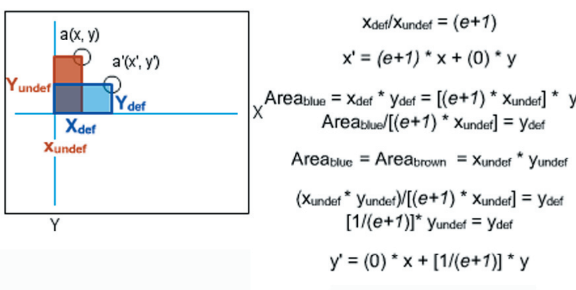

Figure 6. A Flash 5 animation illustrating the key elements in the derivation of the transformation equation for two dimensional pure shear. Select sequentially the items in the Show Geometrical Elements drop-down list. To see the equations for the geometrical elements identified in the resulting animation, select the item from the Show Equation For drop-down list. (**Select image for interactive HTML version**)

From equation (12) and Figure 6 we know that x_{def} = [$(e+1) * x_{undef}$]. Substituting for x_{def} in equation (15) and canceling similar terms yields

$$[1/(e+1)] * y_{undef} = y_{def} \quad (16)$$

Finally, we note that y_{def} is simply y' and that y_{undef} is y. Hence, the coordinate transformation equation for y' is

$$y' = (0 * x) + [1/(e+1)) * y] \quad (17)$$

Equations (12) and (17) also can be written in matrix format as follows:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} (e+1) & 0 \\ 0 & 1/(e+1) \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix} \quad (18)$$

Strain Machine

The Shockwave version of Strain Machine is provided as Figure 7. To fruitfully gain from the following material the reader should have either Figure 7 or the downloaded executable version of Strain Machine opened on their computer so they can perform the various procedures outlined in the remaining portions of this paper.

The first step in any strain experiment is to plot a circle by clicking on the Plot Circle button (Fig. 7). This step produces a circle with a radius of 4 units. Thirty six points along the circumference of the circle are shown as squares of different color. When the mouse pointer is moved over the strain grid, it changes from an arrow to a cross (Fig. 7).

The second step is to select to what kind of an experiment to conduct by clicking on either the simple or pure shear radio button (Fig. 7). In addition, the user must move the sliders to some preferred value of stretch ($e + 1$) and gamma (γ) (Fig. 7).

The third and final step of an experiment is to apply an increment of strain by clicking on the Show Strain button (Fig. 7). After each increment of applied strain, displacement paths for the 36 points are shown, and as a result, vector-displacement maps can be prepared easily from print outs of a given experiment. After each increment of strain, the user can determine the stretch value of a ray extending from the center of the original circle to any of the 36 points lying along the circumference of the plotted ellipse by simply moving the mouse cursor over the point of interest (Fig. 7). The coordinates of the selected point of interest are displayed in two blue boxes on the left hand side of the program's window (Fig. 7). A useful exercise is to verify the stretch value through the use of the Pythagorean Theorem. At any time during the experiment, the user can change the values of ($e + 1$) or gamma (γ), and can switch from pure to simple shear or visa versa. Thus, complex strain paths can be modeled.

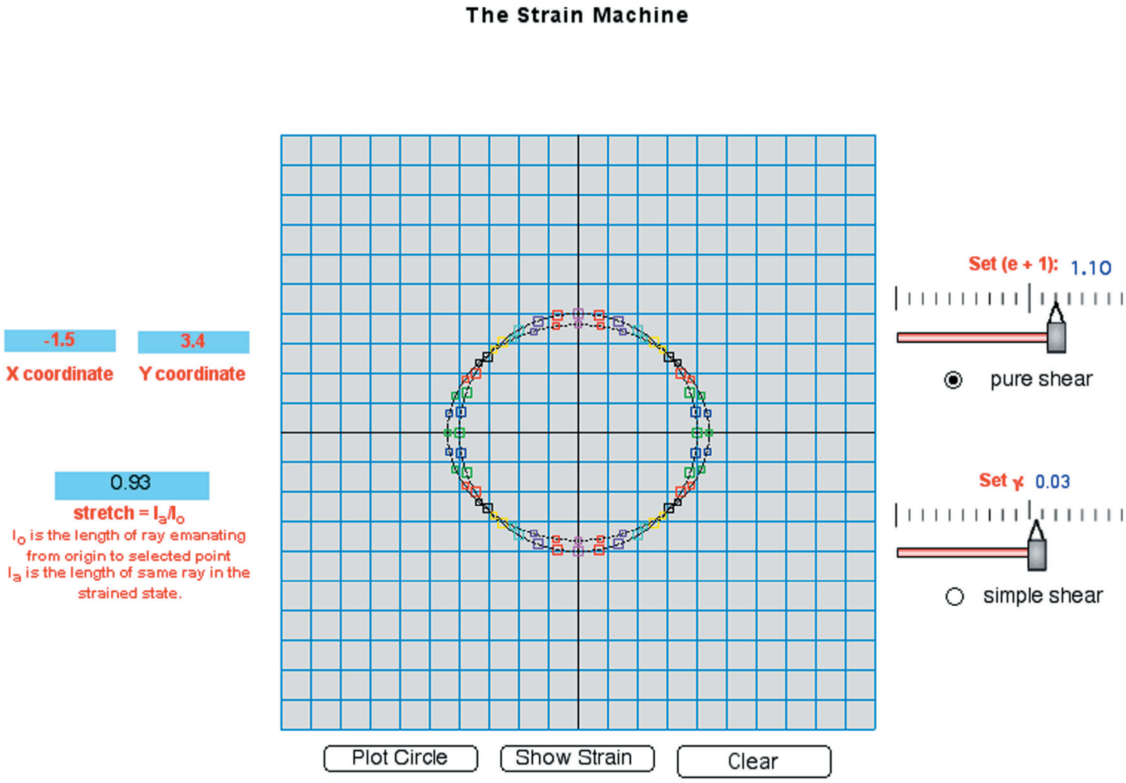

Figure 7. Shockwave Version of Strain Machine. An executable version of Strain Machine with enhanced functionality (e.g., printing capabilities) is available at the author's web site. (Interactive HTML Version: **http://www.virtualexplorer.com.au/2002R/Bobyarchick/Girty/figure7Girty.dcr**)

Before conducting a new experiment the user must first clear the strain grid by clicking on the Clear button (Fig. 7).

Experimenting in Strain Machine

The Exercise

The following exercise was developed so that students can lean the key components of pure and simple shear through the act of discovery and experimentation.

The exercise consists of three parts. In part I students plot an initial circle and then calculate its area (i.e., 3.14 * r^2 where $r = 4$ units). An increment of pure or simple shear is then applied to the plotted circle with γ set at 0.3 and ($e + 1$) set at 1.10. Regardless of which type of strain is selected after application of one increment of strain the initial circle is transformed into the strain ellipse (Fig. 7) with major and minor axes equal to the principal strain axes X and Z. The intermediate strain axis, Y, lies perpendicular to the XZ plane (i.e., the computer screen).

In part II of the exercise, students perform both progressive simple and pure shear experiments applying seven increments of strain utilizing the same settings for ($e + 1$) and γ. Progressive strain refers to the non-rigid motion of a distorting body. During the experiment with Strain Machine it is assumed that the seven increments of strain are applied one after another without any significant time delay between each increment. Hence, motion is approximately steady.

Following application of seven increments of progressive strain students determine the area of the final strain ellipse. The area of an ellipse is πab, where a is the length of the semimajor axis and b is the length of the semiminor axis. All calculations are rounded to two significant figures.

Finally, print outs of the final plot are made and vector-displacement maps of each experiment are prepared (Fig. 8).

In part III, students utilize the data that they have gathered to address the following eight questions.

(1) What are the orientations of the principal strain axes X and Z after each increment of simple and pure shear?

(2) The incremental strain ellipse is characteristic of the strain transformation equation for a given ($e+1$) and γ. Finite strain refers to the total strain that has accumulated over a specified interval of time. How would you characterize the incremental strain ellipse and the finite strains that accumulated during your experiments of progressive simple and pure shear?

(3) If during an experiment the principal axes of a series of finite strains are parallel to the principal axes of incremental strain, then the experiment is an example of coaxial progressive strain. On the other hand if the principal axes of a series of finite strains are not parallel to the principal axes of incremental strain, then the experiment exemplifies non-coaxial progressive strain. Which of the two experiments (progressive simple or progressive pure shear) is an example of coaxial strain and which is an example of non-coaxial strain?

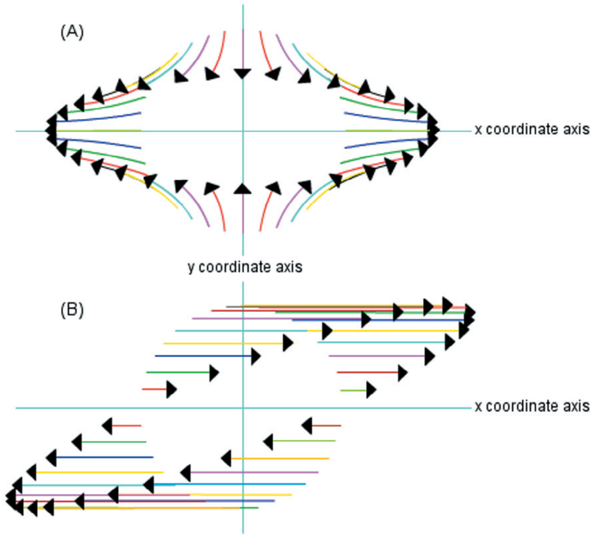

Figure 8. Vector-displacement maps based on 7 strain increments with (e+1) = 1.1 and g = 0.3. (A) Pure shear. (B) Simple shear. Click on the Show Initial Circle and Strain Ellipse button to gain access to animations of particle displacement paths. Note the different paths that particles travel as they are displaced from their positions on the circumference of the circle to their positions on the strain ellipse. (**Select image for interactive HTML version**)

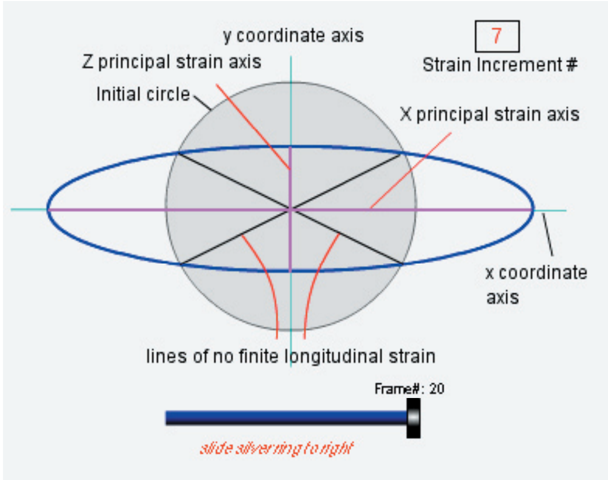

Figure 9. To observe the positions of lines of no finite longitudinal strain during seven increments of pure shear slide the silver ring to the right or click on the Play Movie button. Note that during the experiment that the X and Z principal strain axes lengthened and shortened, but otherwise remained stationary. Any material line whose orientation lies with the pie-shaped segment bisected by Z has been shortened while those with orientations lying within the pie-shaped segment bisected by X have been lengthened. (**Select image for interactive HTML version**)

(4) Is strain isochoric during progressive simple and pure shear?

(5) What do the vector-displacement maps tell you about the displacement paths of material points during progressive simple and pure shear?

(6) Material lines that have not undergone a net change in length are said to be lines of no finite longitudinal strain (i.e., stretch = 1.0) (e.g., Ragan, 1985). Hence, any ray with one end point at the center of the initial circle, and the other end point lying at the same location on the circumference of the initial circle and on the strain ellipse is a line of no finite longitudinal strain. For each increment of pure and simple shear how many material lines exhibit no finite longitudinal strain? What are the orientations of the X and Z principal strain axes relative to the lines of no finite longitudinal strain?

(7) The set of lines of no finite longitudinal strain subdivide the strain ellipse into four wedge or pie-shaped segments. During the experiments what happens to material lines that are contained within each of these four segments?

(8) What happens to the orientations of material lines that are parallel to the principal strain axes X and Z during progressive simple and pure shear?

The Results

From the above experiment students should be able to discover the following key components of strain.

(1) For pure and simple shear applying the transformation equation to a set of coordinates defining a circle produces the strain ellipse (Figs. 8, 9, and 10).

(2) Both simple and pure shear are two end members of plane strain, and are therefore constant area (or volume) distortions that can be described mathematically by a transformation equation. Under such conditions $\pi r^2 = \pi ab$.

(3) Progressive simple and pure shear are a series of strain events, each event being an addition to a growing distortion. Such a process can be modeled for n steps, by first applying the transformation equation to the coordinates of a circle to produce the incremental strain ellipse (Figs. 9, 10). The incremental strain ellipse is representative of the applied strain transformation equation with $(e+1) = 1.10$ and $\gamma = 0.3$. Applying the same transformation equation to the coordinates of the first through nth ellipses produces the second through nth steps (Figs. 9, 10). Thus, after completion of the first step what is displayed in Strain Machine is the finite and not the incremental strain. Though incremental strain was kept constant during the above experiment, it is important to realize that this does not need to be the case.

(4) Tracking the orientations of the principal strain axes during progressive simple and pure shear reveals that simple shear is non-coaxial while pure shear is coaxial (Figs. 9, 10).

(5) Vector-displacement maps for pure shear reveal that during progressive strain, all particles, with the exception of those oriented parallel to the principal strain axes, follow complex curved paths (Fig. 8A). In contrast, particles located along the principal strain axis X follow a linear path moving outward away from the origin while those located along Z follow a linear path moving inward toward the origin.

(6) During pure shear material lines that are oriented parallel to the Z principal strain axis are shortened, while those oriented parallel to the X principal strain axis are lengthened (i.e., extended) (Fig. 9). In contrast, some

material lines not oriented parallel to the principal strain axes undergo complex distortions, sometimes shortening early in the strain history and then lengthening later on.

(7) Vector-displacement maps for progressive simple shear reveal that the 36 particles lying on the circumference of the initial circle follow simple linear paths that are parallel to the x-coordinate axis, i.e., the plane of shear (Fig. 8B).

(8) During progressive simple shear material lines that are not parallel to the x-coordinate axis, rotate toward it. Depending upon their initial orientation some material lines will initially shorten and then pass through an orientation where their stretch values are 1.0. Subsequent increments of strain will result in these lines elongating. The lengths of material lines that are oriented parallel to the x-coordinate axis do not change during progressive simple shear (Figs. 8B, 10). In contrast, material lines parallel to the principal strain axes X and Z lengthen and shorten respectively while rotating toward the x-coordinate axis (Fig. 10).

(9) Following any increment of pure or simple shear two lines of no finite longitudinal strain can be identified (Figs. 9, 10). These lines divide the strain ellipse into four wedge or pie-shaped segments. The principal strain axes X and Z bisect the two lines of no finite longitudinal strain (Figs. 9, 10). Material lines lying in those segments bisected by the Z principal axes have been shortened while those lying in segments bisected by X have lengthened.

Conclusions

Strain Machine and other interactive illustrations used in this paper were designed with the undergraduate student or professional newly introduced to the concept of strain in mind. Hopefully they will aid such individuals in learning the basics of strain theory. I encourage my colleagues to give Strain Machine and the interactive Flash 5 animations a try. From my experience students will like "playing" and "experimenting" with them and in so doing will be gaining intuition about this core concept of structural geology.

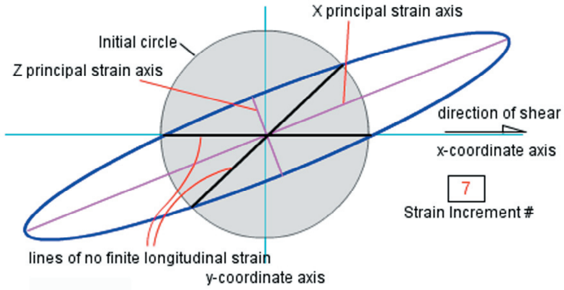

Figure 10. To observe the positions of lines of no finite longitudinal strain during seven increments of simple shear slide the silver ring to the right or click on the Play Movie button. Note that the X and Z principal strain axes have rotated toward the direction of shear while one line of no finite longitudinal strain lies unchanged and parallel to the x-coordinate axis, i.e., the direction of shear. As with figure 9 any material line whose orientation lies with the pie-shaped segment bisected by Z has been shortened while those with orientations lying within the pie-shaped segment bisected by X have been lengthened.

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